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Technical Note

Design of a new scheme to indicate the domain of applicability of near singular approach in 2D BEM

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ABSTRACT

The Gauss–Legendre integration is not appropriate for singular and nearly singular integrations in BEM. In this study, some criteria are introduced for recognizing the nearly singular integrals in integral form of Laplace equation. At first, a criterion is obtained for constant element and consequently higher order elements are investigated. To indicate this near singular approach, there are different formulations amongst which the Romberg method was selected due to its compatibility with analytical integration. The singular integrals were carried out by composing the Romberg method and midpoint rule. The potential functions over geometrically linear BEM elements can be defined in the form of constant, linear or other types of interpolation functions. In those elements, the Gauss–Legendre integration will be accurate, if the source point is placed out of the circle with a diameter equal to element length and its center matched to midpoint of the element. Also, some criteria are obtained for parabolic function of geometry over an element.

1. Introduction

As it is well known, the Gaussian quadratures are the most conventional method for BEM integration. Because it is an accurate method and the calculation time is very short compared to other numerical integration methods. But in the case of singular or nearly singular integrals, the ordinary Gaussian quadrature is not accurate [1].

A number of researches have investigated singular integrals in BEM. A new method, known as direct Gauss quadrature formula, was introduced by Smith [2] for singular integrals. Ozgener and Ozgener [3] verified a newly developed quadrature formula for singular integrals. Sladek and Sladek [4] explained and defined the singularity in BEM and Zisis and Ladopoulos [5] introduced an exact solution for singular integrals in BEM.

Many researchers have focused on nearly singular integrands of integral form of Laplace equation in boundary element method. Ma and Kamiya [6] have introduced a general algorithm for accurate integration of nearly singular integration, known as boundary layer effect in BEM. Niu et al. [7,8] have focused on the analytical integration of nearly singular integrands for some types of elements of BEM by introducing relative distance.

As mentioned, all the above researches introduced different methods for solving nearly singular problem in BEM. This study attempts to answer the following important question: In what relative position between source point and element we would have a nearly singular integral? In this research, the position of source point relative to element has been determined such that Gauss–Legendre quadrature would be accurate for BEM integrals. Also, the Romberg integration method is used for nearly singular integrals.

2. Gauss–Legendre integration

Gauss–Legendre integration is based on the following equation [9–12]:

$$\int_{-1}^{1} f(x)dx = \sum_{i=1}^{n} w_i f(x_i) \tag{1}$$

In which f is a function with m times (depending on number of Gauss points) differentiable, w_i are weight factors and x_i are Gauss points.

Taylor series around zero (Maclaurin series) can be used for deriving Gauss–Legendre integration parameters [10].

Weight factors and Gauss points of n points Gauss integration are obtained by applying some usual formulas (for example Legendre's polynomials), which can be found in many numerical analysis books. Also, the error could be calculated by following equation [9–12]

$$E = \frac{2^{2n+1} \cdot (n!)^4}{(2n+1)((2n)!)^3} f^{(2n)}(\xi) \tag{2}$$

In which n is the number of points of Gauss integration.

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