Engineering Analysis with Boundary Elements 35 (2011) 868–878

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A meshless Galerkin least-square method for the Helmholtz equation

Zeng He [a,b], Peng Li [a,b,*], Gaoyu Zhao [a,b], Han Chen [a]

[a] Department of Mechanics, Huazhong University of Science and Technology, Wuhan 430074, China
[b] Hubei Key Laboratory for Engineering Structural Analysis and Safety Assessment, Wuhan 430074, China

ARTICLE INFO

Article history:
Received 3 October 2010
Accepted 24 January 2011
Available online 2 March 2011

Keywords:
Acoustics
Meshless
Least-square
Dispersion
Helmholtz equation

ABSTRACT

The Helmholtz equation always suffers the so-called 'pollution effect', which is directly related to the dispersion for high wavenumber. The element-free Galerkin method (EFGM) has been successfully applied to acoustic problems and significantly reduced the dispersion error. Unfortunately, it is computationally expensive. In this paper, a two-dimensional (2-D) dispersion analysis is performed on the meshless Galerkin least-square (MGLS) method. This method is based on the EFGM at the domain boundary and the least-square method in the interior. Numerical examples on an L-shaped cavity demonstrate that while retaining the accuracy of the EFGM, the computational cost can be significantly reduced.

1. Introduction

The Helmholtz equation is an elliptic partial differential equation, which is important in a variety of applications involving time-harmonic wave propagation phenomena such as acoustic cavity and radiation wave. Accurate and efficient numerical simulation of acoustic problems governed by the Helmholtz equation is still an open problem especially for medium frequencies. By using the standard finite element method (FEM), the numerical phase accuracy deteriorate rapidly as the wavenumber, while the boundary element method (BEM) suffers from high computational costs because of the full and non-symmetric algebraic equations to be solved.

In order to depress the dispersion for high wavenumber, highly refined finite element meshes (*h*-FEM) or higher orders of polynomial approximation (*p*-FEM) are required, and the *hp*-FEM [1] seems to give good results. However, to obtain an acceptable level of accuracy, more than ten elements per wavelength are required. For large wavenumber, refining the mesh to maintain this requirement may become prohibitively expensive. Several methods to stabilize the FEM have also been developed, such as the Galerkin least-square (GLS) FEM [2], the quasi-stabilized FEM (QSFEM) [3], the residual-free FEM (RFEM) [4] and the residual-based FEM [5] with applications to the Helmholtz problem. A review of these methods can be seen in [6,7]. However, none of them eliminates the dispersion.

Meshless methods have several advantages over the classical mesh-based methods. In the case of the Helmholtz equation, it has already been shown that the EFGM [8,9], the multiple-scale reproducing kernel particle method (RKPM) [10] and the so-called radial point interpolation method (RPIM) [11], give very accurate results for interior Helmholtz problems. Unfortunately in order to ensure their accuracy, delicate background cells and a large number of quadrature points have to be used for the global numerical integration for the Galerkin method (the EFGM, the RKPM and the RPIM), which dramatically increases the computational cost. Recently two methods based on the method of fundamental solutions (MFS) [12] and the boundary knot method (BKM) [13] have been extended to Helmholtz-type equations. Both the two methods, however, need to use the inner nodes for inhomogeneous problems to guarantee the stability and accuracy of the solution. More recently the boundary-node method (BNM) [14] has been applied to Helmholtz problems which showed high convergence rates and high accuracy. However, it is difficult to satisfy the boundary conditions accurately in BNM. This makes it computationally much more expensive than the BEM [15].

A new meshless weighted least-square (MWLS) method was proposed to solve problems of elastostatics [16], wave propagation and large deformation [17], and the advantages of better accuracy, high efficiency and fast convergence were demonstrated. More recently it has been successfully extended to steady and unsteady state heat conduction problems [18,19]. Pan et al. [20] found that the MWLS method is sensitive to boundary conditions. He developed the meshless Galerkin least-square (MGLS) method, with the Galerkin method on the boundary and the least-square method in the interior domain, and applied it to elasticity problems. An alternative scheme is to use FEM interpolants in the boundary [21]. In this paper, the MGLS method is extended to acoustic problems, with a focus on the 2-D dispersion effect for the Helmholtz equation.

The paper is organized as follows. Section 2 presents the strong form of the general acoustic problem. Section 3 gives a brief

* Corresponding author at: Department of Mechanics, School of Civil Engineering and Mechanics, Huazhong University of Science & Technology, Wuhan 430074, China. Tel.: +86 27 87543338; fax: +86 27 87543038.
E-mail address: hnlipeng@gmail.com (P. Li).

0955-7997/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.enganabound.2011.01.010