Engineering Analysis with Boundary Elements 35 (2011) 895–907

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Comparison among three boundary element methods for torsion problems: CPM, CVBEM, LEM

Giorgio Barone, Antonina Pirrotta, Roberta Santoro*

Dipartimento di Ingegneria Strutturale Aerospaziale e Geotecnica (DISAG), Università degli Studi di Palermo, Viale delle Scienze 90128, Palermo, Italy

A R T I C L E I N F O

Article history:
Received 16 June 2010
Accepted 20 January 2011
Available online 30 March 2011

Keywords:
Boundary Element Methods
Complex analysis
Torsion

A B S T R A C T

This paper provides solutions for De Saint-Venant torsion problem on a beam with arbitrary and uniform cross-section. In particular three methods framed into complex analysis have been considered: Complex Polynomial Method (CPM), Complex Variable Boundary Element Method (CVBEM) and Line Element-less Method (LEM), recently proposed. CPM involves the expansion of a complex potential in Taylor series, computing the unknown coefficients by means of collocation points on the boundary. CVBEM takes advantage of Cauchy's integral formula that returns the solution of Laplace equation when mixed boundary conditions on both real and imaginary parts of the complex potential are known. LEM introduces the expansion in the double-ended Laurent series involving harmonic polynomials, proposing an element-free weak form procedure, by imposing that the square of the net flux of the shear stress across the border is minimized with respect to the series coefficients. These methods have been compared with respect to numerical efficiency and accuracy. Numerical results have been correlated with analytical and approximate solutions that can be already found in literature.

1. Introduction

Laplace equation is one of the most important partial differential equation governing many problems in mathematical physics such as heat flow, electrostatics, fluid flow, gravitational field, magnetism, diffusion, elasticity and current flow. Problems governed by Laplace equation are studied by means of the so-called potential theory and its solutions, whose second partial derivatives are continuous, are called harmonic functions.

Use of complex analysis for developing approximations for two-dimensional potential problems is an efficient tool for numerical analysis of problems related to systems governed by the Laplace equation.

Two main advantages are reached by use of complex analysis: a technical one and a physical one. Technically, it is possible to characterize the solution for two conjugate functions at once; physically, the real and the imaginary part are orthogonal functions and have a general meaning of potential and stream functions. For canonical domains (circle, rectangle) or for particular domains the Laplace equation can be solved using analytical methods and conformal mapping [1]. When dealing with domains of a complex shape, numerical methods such as Boundary Difference Method [2] and Finite Element Method [3–5] are widely used. However, both methods need a discretization of the whole domain that is time consuming. In this context Complex Polynomial Method (CPM) [6–7], Complex Variable Boundary Element Method (CVBEM) [8–10] and Line Element-less Method (LEM) [11] can be considered as efficient tools for the numerical analysis of the Laplace equation solution improving computational facility by use of complex analysis. Moreover, LEM does not require any discretization, neither of the boundary nor the domain, as it will be detailed in the next sections.

This paper aims at comparing these three numerical methods for the solution of torsion problems in De Saint-Venant cylinder with arbitrary, but uniform, cross-section, considering both smooth and sharp corners profiles. At first shape sections whose exact solution is known (circle, ellipse, equilateral triangle) will be considered, so that the advantages of the methods may be immediately captured. Then more generic sections will be analyzed.

2. Complex analysis to potential theory

As already stressed in the Introduction, many physical problems of interest are mathematically described by the two-dimensional Laplace equation [1,12–13] as follows:

$$\nabla^2 U(\hat{z}) = \frac{\partial^2 U(\hat{z})}{\partial x^2} + \frac{\partial^2 U(\hat{z})}{\partial y^2} = 0 \text{ in } A \quad (1)$$

being

$$U(\hat{z}) = \omega(x,y) + i\varphi(x,y) \quad (2)$$

* Corresponding author. Tel.: +390916568406.
E-mail addresses: giorgio.barone@unipa.it (G. Barone), antonina.pirrotta@unipa.it (A. Pirrotta), roberta.santoro@unipa.it (R. Santoro).

0955-7997/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.enganabound.2011.02.003