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An efficient implementation of the generalized minimum residual algorithm with a new preconditioner for the boundary element method

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A B S T R A C T

Despite the numerical stability and robustness of the generalized minimum residual algorithm, it still suffers from slow convergence rate and unexpected breakdown when applied to the boundary element method, even with the conventional preconditioners. To address these problems, we have devised a new preconditioner by combining the partial pivot method and diagonal scaling preconditioner with use of the selective reorthogonalization criterion. We examine the performance of these implementations through three numerical examples having a simple-domain, a multi-domain and a multiply-connected domain. The results of the numerical analyses confirm that the selective reorthogonalization criterion can retain the orthogonality of the basis vectors with a small number of reorthogonalizations and that the proposed preconditioner improve the computational efficiency.

1. Introduction

Traditionally, direct methods (e.g., variants of the Gauss elimination method) have been favored for solving linear algebraic system of equations of the boundary element method (BEM) over iterative methods due to their accuracy and numerical stability. However, it is too expensive to solve high degrees of freedom (DOF) problems routinely encountered in many real engineering applications by the direct solver, whose computational cost is proportional to $O(n^3)$, where n is the number of DOF. In contrast, the computational cost of iterative methods is approximately only $O(n^2)$ when the coefficient matrix of system of equations is well conditioned [1,2]. Besides, successful application of iterative methods in the field of the finite element analysis (FEA) called the attention of the boundary element scientific community to the necessity of new efficient alternative solution techniques [3]. Consequently, there have been considerable efforts to develop an iterative solver that is computationally efficient and less prone to round-off errors than direct methods for a BEM computer program.

The first approach to employ iterative methods in the field of boundary element analysis was made to solve the two-dimensional Laplace equation by Mullen and Rencis [4]. They examined various iterative methods, including those valid only when the coefficient matrix of system of equations is positive symmetric or at least diagonally dominant. However, because the coefficient matrix of BEM system of equations is dense, asymmetric and indefinite as well as lacking diagonal dominance [3], a majority of iterative methods diverge or present very slow convergence rates [5]. Furthermore, the convergence rate of iterative methods, valid for the BEM, can be slowed if one applies sub-structuring techniques, nodal coordinate transformations, mixed boundary conditions or contact conditions [1–3,6]. In this context, Urekew and Rencis [7] proposed an algorithm to transform the coefficient matrix of the BEM system of equations into an equivalent diagonally dominant matrix and improve the condition of the coefficient matrix. Even though this transformation algorithm guarantees the convergence of the iterative methods classified as stationary methods, it has been used only as a research tool because it spends more computational resources than the direct solver.

The generalized minimum residual algorithm (GMRES) has been recognized as the best suitable iterative method for the asymmetric and dense coefficient matrix due to its numerical stability and robustness [8,9] since being developed by Saad and Schultz [10]. Nonetheless, it still suffers from slow convergence rates [2,11,12] and unexpected breakdown in a practical engineering problem of the BEM [13] because the coefficient matrix of the BEM is too ill-conditioned. Additionally, the restarted version of GMRES algorithm, which has been considered as the standard remedy to the unexpected breakdown and the large memory requirements, can still make the convergence rates worse [14] and fail to converge by the complete stagnation [15]. To fix these problems of the GMRES algorithm there have been two approaches. The first one is the full reorthogonalization method which has been used to overcome the unexpected breakdown of

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