



Least squares stochastic Boundary Element Method

Marcin Kamiński*

Department of Structural Mechanics, Faculty of Civil Engineering, Architecture and Environmental Engineering, Technical University of Lodz, Al. Politechniki 6, 90-924 Łódź, Poland

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ABSTRACT

The main issue in this paper is mathematical formulation and computational implementation of the stochastic Boundary Element Method based on the generalized stochastic perturbation technique. The key feature is a replacement of the given order polynomial response function with the least squares method leading to a numerical determination of this response function. This new approach minimizes the approximation error during the recovery of the structural response indexed with the random input parameter, which is a decisive factor for the entire stochastic method accuracy; contrary to some lower order techniques, numerical implementation of up to the fourth order probabilistic moments is displayed. Computational experiments obey both analyses for the homogeneous and heterogeneous structures with Gaussian random material parameters and also some comparison against the Monte-Carlo simulation and analytical results.

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1. Introduction

Structural problems with random data appear frequently in engineering practice, especially in all those cases, where some parameters are taken from laboratory experiments or from *in situ* measurements. Statistical estimators of these parameters influence the final random response of the engineering system being modeled. In deterministic analysis instead of full statistical information the expected values are included into the computational model. As it is widely known, there is a variety of stochastic theoretical and computational methods that offer full correspondence between random input data and the modeling options [1]. Besides the oldest Monte-Carlo simulation techniques, the spectral, the perturbation, some symbolic as well as the random matrix methodologies are employed to determine the influence of input probabilistic characteristics on the final random behavior of an engineering system. Their implementations in conjunction with the Boundary Element Method (BEM) are available in mainly geotechnical problems, where parameter random dispersion in civil engineering seems to be the largest one, for groundwater [2–4], seawater [5] and porous media flows [6], in heat conduction and transfer issues [7,8] or even for some elastodynamics [9] and wave propagation problems [10,11]. It should be mentioned that the SBEM analyses are especially effective in random boundary geometry modeling [12], so that it is used in the problems with random boundary conditions [13–15] or in stochastic shape design sensitivity [16]. The perturbation method is a specific approach,

where both input and output structural parameters are expanded using classical Taylor series with random parameters. This expansion is made around their expectations using the perturbation parameter ε together with the partial derivatives of increasing order calculated with respect to input random parameter. Usually, this perturbation was reduced to the second order only, which excluded numerical analyses of probabilistic problems with the coefficient of variation larger than 0.10. Moreover, the computation of higher than the second probabilistic moments was quite ineffective according to the second order approach.

Therefore, the new version of the generalized, perturbation-based stochastic Boundary Element Method is invented here and tested numerically. This version of SBEM offers both higher perturbations orders and a computation of higher than the second probabilistic moments analogously to the first version based also on the Response Function Method (RFM) [17]. The basic difference is in numerical determination of this response function, which now is carried using the nonlinear least squares fitting. Now it is also possible to optimize the order of the approximating polynomial during the entire symbolic processing in the computer algebra system MAPLE, v. 13. Let us remind here that the order of this approximation was given a priori and corresponded directly to the number of trial points around the mean value of the random input parameter. This order optimization undoubtedly leads in turn to minimization of the approximating error (and effectively speeds up the probabilistic convergence of the method) since apparent linear interrelations do not need to be approximated using 10th order polynomials. Now, the optimal order of the approximating polynomials determined via the least squares method guarantees the perfect matching of the trial set of computational results with the response function. It was not the case of some previous studies,

* Tel./fax: +48 42 6313571, mobile: +48501218678.

E-mail address: Marcin.Kaminski@p.lodz.pl