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# Adaptive methodology for the meshless Galerkin boundary node method

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ABSTRACT

The Galerkin boundary node method (GBNM) is a boundary-type meshless method that combines a variational form of boundary integral formulations for governing equations with the moving least-squares approximations for generation of the trial and test functions. In this paper, a posteriori error estimate and an effective adaptive *h*-refinement procedure are developed in conjunction with the GBNM. The error estimator is based on the difference between numerical solutions obtained using two successive nodal arrangements. The reliability and efficiency of this error estimator and the convergence of this adaptive meshless scheme are verified theoretically. Numerical examples are also given to show the efficiency of the adaptive methodology.



## 1. Introduction

Meshless (or meshfree) methods for numerical solutions of boundary value problems have generated much attention and gained great success in the field of computational science and engineering in the past few decades [1,2]. Although many types of meshless methods have been already proposed, these methods can be classified collectively as the domain-type and the boundary-type. The domain-type meshless methods followed the idea as the finite element method (FEM), in which the problem domain is discretized by nodes. Most of meshless methods like the element free Galerkin (EFG) method [1,2], the reproducing kernel particle method (RKPM) [1], the point interpolation method (PIM) [2], the *h-p* meshless method [3], the generalized FEM [4], the meshless local Petrov–Galerkin (MLPG) method [5], the finite point method (FPM) [6], the finite cloud method (FCM) [7], the smoothed FEM [8] and so on fall into this group. These meshless methods have been achieved remarkable progress in solving a wide range of boundary value problems.

The boundary-type meshless methods are developed by the combination of the meshless idea with boundary integral equations (BIEs), such as the boundary node method (BNM) [9,10], the hybrid boundary node method (HBNM) [11–13], the boundary point interpolation method [14], the boundary element-free method (BEFM) [15], the boundary face method [16,17] and the Galerkin boundary node method (GBNM) [18]. Compared with the domain-type meshless methods, they require only a nodal data structure on the bounding surface of a body whose dimension is less than that of the domain itself. So like the boundary element method (BEM), they are superior in tackling problems dealing with infinite or semi-infinite domains. The meshless local boundary integral equation (LBIE) method [19] is another BIEs-based meshless method, which is equivalent to a sort of MLPG approaches. The LBIE method, however, is not strictly a boundary-type method since it requires computation of integrals over certain surfaces (called $L_s$ in Ref. [19]) that can be regarded as "closure surfaces" of boundary elements.

It is well known that the problem of generating an adaptive mesh refinement process is of practical importance in numerical analysis of partial differential equations. Adaptive procedures are now common tools in science and engineering. In the past 40 years, the research of error estimation and refinement strategies has seen enormous activities in the context of classical numerical methods formulated based on mesh such as the FEM and the BEM. Comprehensive reviews on the development of the corresponding adaptive analysis can be found in open literature such as Refs. [20,21]. The FEM and the BEM depend on the generation of meshes, adapted or not. In some cases, this can be time-consuming and very difficult, especially for adaptive analysis. On the contrary, since no predefined nodal connectivity or mesh is employed in meshless methods for field variable approximation, mesh-related difficulties can be avoided. This prominent feature simplifies significantly the implementation for adaptive meshless schemes, as nodes can be conveniently inserted or removed for the refinement or coarsening procedures.

In recent years, considerable efforts have been done in developing reliable error estimator and effective refinement procedures for adaptive analysis in domain-type meshless methods. In the framework of the *h-p* cloud method, Duarte and Oden [22] have derived a residual based error estimator that involves only the computation of interior residuals and the residual on the

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