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A fast multipole boundary element method based on the improved Burton–Miller formulation for three-dimensional acoustic problems

Shande Li, Qibai Huang *

State Key Laboratory of Digital Manufacturing Equipment and Technology, School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, PR China

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ABSTRACT

A fast multipole boundary element method (FMBEM) based on the improved Burton–Miller formulation is presented in this paper for solving large-scale three-dimensional (3D) acoustic problems. Some improvements can be made for the developed FMBEM. In order to overcome the non-unique problems of the conventional BEM, the FMBEM employs the improved Burton–Miller formulation developed by the authors recently to solve the exterior acoustic problems for all wave numbers. The improved Burton–Miller formulation contains only weakly singular integrals, and avoids the numerical difficulties associated to the evaluation of the hypersingular integral, it leads to the numerical implementations more efficient and straightforward. In this study, the fast multipole method (FMM) and the preconditioned generalized minimum residual method (GMRES) iterative solver are applied to solve system matrix equation. The block diagonal preconditioner needs no extra memory and no extra CPU time in each matrix–vector product. Thus, the overall computational efficiency of the developed FMBEM is further improved. Numerical examples clearly demonstrate the accuracy, efficiency and applicability of the FMBEM based on improved Burton–Miller formulation for large-scale acoustic problems.

1. Introduction

Boundary element method (BEM) has long been considered as a very promising technique in acoustics, since it allows the simulation of fields in unbounded domains and automatically satisfies the radiation condition at infinity. Only the boundary of the sound structure must be discretized rather than the entire domain, which implies a very low cost for mesh generation and preprocessing [1–3]. Despite these advantages, the BEM, however, has a serious problem. The conventional BEM leads to linear systems with dense, non-symmetric matrices. For an acoustic problem with N unknowns, computing the coefficient matrices requires $O(N^2)$ operations, and solving the system of equations requires another $O(N^2)$ operations with iterative solvers such as the generalized minimum residual method (GMRES) [4]. As a result, the conventional BEM is prohibitively expensive for large-scale engineering problems. Many research works have been devoted in improving the applicability of the BEM for large-scale problems, such as iterative solvers, parallel computing or sub-domain techniques. Still the BEM has been restricted to solve relatively small-size problems with unknowns about 10 000 on a personal computer (PC), until the development of the fast multipole method (FMM) [5–7], drastically changed the circumstances around the BEM.

In the mid-1980s, FMM was initially introduced by Rokhlin [5] as a fast solution method for boundary integral equations, and then was further developed by Greengard [6,7] for N-body problems. Later a new version of the FMM was developed by Greengard and Rokhlin [8], the new FMM used the intermediate exponential expansions to accelerate the Laplace equation. The key idea of the FMM is to combine the effect of sources far away from a collocation point in a far-field term using the multipole expansion whereas for nearby sources the conventional BEM evaluations are used. Employing the FMM and iterative solvers such as GMRES, the CPU time and memory requirement can be drastically reduced from $O(N^2)$ to $O(N)$. Thus this method enables many researchers apply the BEM to really solve large-scale problems. The FMM has been intensively studied and extended to solution of various scientific fields, including those arising from the Helmholtz, Maxwell and elasticity equations [9–18]. A comprehensive review of the fast multipole method can be found in Ref. [12].

The fast multipole BEM (FMBEM) has been also studied in the field of acoustics, not only for developing the theory, but also for practical applications. From 1990s, the FMM began to be applied to acoustic problems [13–18], Rokhlin [15] proposed diagonal form of the translation matrices for high frequency Helmholtz

* Corresponding author. Tel.: +86 27 87557664; fax: +86 27 87544175.
E-mail addresses: lishande@gmail.com (S. Li), qbhuang@mail.hust.edu.cn (Q. Huang).

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