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Technical Note

A boundary-integral algorithm for adaptive motion planning

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ABSTRACT

An algorithm is developed for adaptively computing the path of an object in a domain with arbitrary geometry in the presence of stationary or arbitrarily moving obstacles. The methodology hinges on an analogy with heat transfer where hard walls and obstacles are cold surfaces while targeted positions such as doorways or laboratory benches are hot surfaces. At any instant, the navigator moves in a direction that maximizes the rate of inward heat transport so as to get warmer. A central part of the algorithm is the fast solution of the heat transfer problem using a boundary-element method. Illustrative examples are provided for stationary and moving obstacles in sample domain geometries.

1. Introduction

Motion planning describes the process of predicting or finding the navigation path of an object in a domain with prescribed geometry in the presence or absence or moving obstacles, subject to constraints. The object can be identified with a wheelchair moving in a furnished room, an industrial robot navigating from bench to bench in a laboratory, a surgical instrument operating by remote controls, or an animated body in computer graphics. Applications can be found in the fields of robotics, medical instrumentation and remote surgery, computer animation, building and landscape architecture.

A framework for computing smooth trajectories based on a potential field formulation was proposed by Khatib [2]. The main idea is that an object should move toward the negative of the gradient of a craftily constructed potential function imparting a force. Obstacles exert a repelling force and the target exerts an attractive force leading to a successful navigation path that is free of collisions. To be useful, a potential field must be free of local extrema with vanishing gradient leading to false targets. To preclude this possibility, a harmonic potential field that satisfies Laplace's equation was advocated by Connolly et al. [1]. The properties of harmonic functions guarantee that false targets inside the domain of motion do not arise. Similar implementations were proposed by other authors [3,6].

The potential field theory can be used for stationary or moving obstacles in a dynamic environment, provided that the field function is reconstructed at every instant by solving, for example, Laplace's equation. In practice, the field can be generated by finite-difference methods for simple geometries or finite-element methods for complicated geometries. One limitation of the basic algorithm is that the size of a moving object is assumed to be infinitesimal, so that even though the centerpoint of the object follows a successful navigation path, the perimeter of the object neither bumps into obstacles nor intercepts walls. Pimenta et al. [4] recently presented an improved implementation where a finite object moves along the gradient of a function in two-dimensional space satisfying Laplace's equation in three-dimensional configuration space.

In this paper, a new interpretation and an improved implementation of the potential field theory are proposed based on an analogy with a heat transfer problem. Hard walls and obstacles in a navigation domain are colder than a moving object, and the targeted position, such as a doorway or a laboratory bench, is warmer than the object. At any instant, the object moves toward a warm target in a direction that maximizes the rate of inward rate of heat transport while avoiding cold walls. The heat transport problem governed by Laplace's equation is solved subject to the Dirichlet boundary condition specifying the object and wall temperatures. False paths are prevented by sufficiently increasing the temperature of the target and undesirable walls are avoided by making them sufficiently cold. The centerpiece of the algorithm is the efficient solution of the heat transfer problem using a boundary-element method.

2. Problem statement and mathematical formulation

We want to compute the smooth path of an object moving from an initial to a targeted position representing, for example, the door of a room, in the presence of stationary or moving obstacles, as shown in Fig. 1. Given the instantaneous object position and boundary configuration, we solve Laplace's equation for the temperature at any instant, $\nabla^2 T = 0$, with appropriate Dirichlet boundary conditions. The temperature is set to the

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