Engineering Analysis with Boundary Elements 35 (2011) 114–122

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Three-dimensional boundary singularity method for partial-slip flows

Shunliu Zhao [1], Alex Povitsky *

Department of Mechanical Engineering, Auburn Science and Engineering Center, The University of Akron, Akron, OH 44325-3903, USA

ARTICLE INFO

Article history:
Received 2 March 2009
Accepted 23 June 2010
Available online 21 August 2010

Keywords:
Boundary singularity method
Method of fundamental solutions
Three-dimensional
Knudsen number
Partial-slip boundary conditions
Stokeslets

ABSTRACT

This study extends a 3-D method of the fundamental solutions (MFS) to Stokes flows with Knudsen numbers corresponding to partial-slip flows interior and exterior to spherical boundaries. The study focuses on the distribution of the singularities outside the fluid flow domain. Local spherical coordinates systems are thus introduced to accommodate the application of the method to flows with partial-slip boundary conditions, where velocities tangent to solid walls are proportional to shear rates at surfaces. The singularities are subject to variation in location and number to investigate their impacts on numerical accuracy. For the flow about a single sphere, it is shown that the numerical accuracy improves when the singularities move towards the sphere center. When the singularities are located too far away from the observation points toward the center of sphere, the solution of the linear system fails. The reasons that cause the failure are explored and optimal location of singularities is found. The flow between two concentric spheres is used for further validation of the developed method for a combination of convex and concave surfaces. Finally the application of the method to flows about two separately spaced spheres is presented. Numerical results obtained compare favorably with analytical solutions for presented test cases. It is shown that a moderate number of singularities can be used in combination with a proper location of singularities to achieve a prescribed accuracy.

1. Introduction

This study was motivated by the need for efficient numerical modeling of low-Reynolds-number ($Re<1$) micro- and nano-scale fluid flows in the partial-slip flow regime ($0.01<Kn\le 0.1$), where Kn is the Knudsen number and defined as the ratio of the molecular mean free path over a characteristic length [1]. In the continuum flow regime, boundary singularity methods (BSM) are efficient and have been popular for the Stokes flows, e.g., cavity flows [2], flows past or due to the motion of solid particles [3,4], spiral swimming flows [5,6], and movement of spherical particles in capillaries [7]. In [8], the BSM was extended to the partial-slip flow regime about 2-D filtration flows. Coupled with the direct simulation Monte Carlo method, the BSM with proposed optimal location of singularities was applied to the transition molecular-to-continuum flow regime [9]. Preliminary results describing application of the method for sample 3-D partial-slip flows about ensemble of spherical particles are presented in [10]. The current study focuses on the development of the 3-D BSM for partial-slip Stokes flows.

The idea underlying the BSM is to construct the solution by superposing the singularities (fundamental solutions) and then find the strength of singularities by satisfying the boundary conditions. For the Stokes equations, singularities corresponding to various building block flows are available, including Stokeslet, potential doublet, stresslet, rotlet, Stokeson, roton and stresson [11,12]. It is worth mentioning that the BSM, also referred to as the method of fundamental solutions (MFS), F-Trefftz method, source method, etc. [13], has been widely used in the numerical solutions for the Laplace, Poisson, bi-harmonic, Helmholtz and diffusion equations [2,14]. A comprehensive review of the BSM about elliptic boundary value problems and the advantages of the BSM over BEM were presented in [14].

A Stokeslet fundamental solution is a solution of the Stokes system of equation with a delta function as a non-homogeneous term and consequently it is an exact solution of the Stokes system everywhere except at the source point. Therefore a solution of Stokeslet flow problem can be represented by a discrete distribution of Stokeslet only when the source points are located outside the problem domain, including its boundaries. Substantial gain in accuracy can be achieved by optimizing the locations of singularities [14,3,15–17]. The BSM with least square fitting for adaptive or moving singularities was investigated in [3,15], where the final location and strength of the singularities were determined by an iteration procedure. Adaptive methods that determine both strength and location of singularities need to solve non-linear systems of equations iteratively (as opposed to

* Corresponding author. Tel.: +1 330 972 2685; fax: +1 330 972 6027.
E-mail address: povitsky@uakron.edu (A. Povitsky).
[1] Present address: Department of Civil and Environmental Engineering, Carleton University, Ottawa, Ont., Canada K1S 5B6.

0955-7997/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.enganabound.2010.06.024