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Simulation of crack propagation in anisotropic structures using the boundary element shape sensitivities and optimisation techniques

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A B S T R A C T

Using the boundary element shape sensitivities of multi-region domains coupled with an optimisation algorithm and an automatic mesh generator, the crack kink angle and propagation path in anisotropic elastic solids are predicted. The maximum strain energy release rate criterion, best suited for the composite structures, has been employed. In contrast to the *J*-integral method, which would require the computation of stresses and strains at a series of internal points, here by direct differentiation of the structural response the strain energy release rates at the existing crack tip and new cracks for the period of crack growth are determined. The length of each kinked crack is treated as the shape design variable. The shape variable is then associated with the coordinates of a series of boundary nodes located on the new crack surface. Thus, the relevant velocity terms are applied together in the sensitivity analysis with respect to that variable to determine the energy release rate, which is the derivative of the total strain energy with respect to the crack length extension. Wherever possible the results are compared with the existing experimental, analytical and/or numerical results reported in the literature, in which good agreement is observed. It is shown that the present method is computationally more accurate and efficient. Two example problems with different anisotropic material properties are presented to validate the applications of this formulation. The results show that material anisotropy has a profound influence on the crack propagation of composites.

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1. Introduction

Laminated composites are gaining importance in aerospace structural applications because of their attractive performance characteristics such as high strength-to-weight ratios, high stiffness-to-weight ratios, superior fatigue properties, and high corrosion resistance. Aircraft structures are prone to cracking due to wear and tear, especially when they are used beyond their fatigue life. The tensile strengths of composite laminates are significantly reduced when stress concentrations such as cracks and cutouts are present. The finite element (FE) and boundary element (BE) are the most extensively used methods for the design and strength analysis of anisotropic structures, particularly, for solving fracture mechanics problems. However, the BEM, being a boundary-oriented technique, can overcome a number of the difficulties associated with its main rival, the FEM. Early contribution to the development of the boundary element method (BEM) for cracked anisotropic plates belongs to Snyder and Cruse [1].

The mixed-mode crack problems in two-dimensional linear elasticity can be characterised by a pair of stress intensity factors (SIF's). For isotropic materials, SIFs have been used to predict fatigue crack growth and fracture. Generally, the most popular method to determine the SIFs of a cracked body is the *J*-integral method [2]. The *J*-integral is path-independent for all integral paths surrounding the crack tip. Using this method requires the computation of stresses and strains at a series of internal points around the crack, for evaluation of the path-independent integrals, which is obviously time consuming.

For elastic problems the J_1-integral is the energy release rate per unit extension of the crack. In conjunction with the finite element method (FEM) or BEM, it is possible to use the shape sensitivity analysis to directly evaluate the sensitivities of the total strain energy where the crack length is being treated as the shape variable.

In a recent study by the author [3] the application of the boundary element shape sensitivity for the analysis of two-dimensional anisotropic bodies with cracks of arbitrary shapes was presented. The design sensitivity analysis of multi-region domains with anisotropic material properties was carried out by direct differentiation of the structural response. The length of the crack of an arbitrary shape was treated as the shape variable. The shape variable was then associated with the coordinates of a series of boundary nodes located on the crack surfaces. Thus, the relevant velocity terms were applied together in the sensitivity analysis with respect to that variable to determine the derivatives of displacements, stresses and the elastic compliance of the

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