Engineering Analysis with Boundary Elements 35 (2011) 1303–1317

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A novel variable power singular element in G space with strain smoothing for bi-material fracture analyses

L. Chen [a,b,*], G.R. Liu [c], K. Zeng [b], J. Zhang [d]

[a] Institute of High Performance Computing, Singapore 138632, Singapore
[b] Department of Mechanical Engineering, National University of Singapore, 9 Engineering Drive 1, Singapore 117576, Singapore
[c] University of Cincinnati, Cincinnati, OH 45221-0070, USA
[d] Department of Civil and Environmental Engineering, National University of Singapore, 1 Engineering Drive 2, Singapore 117576, Singapore

ARTICLE INFO

Article history:
Received 14 January 2011
Accepted 14 June 2011

Keywords:
Singular element
Numerical method
G space
Strain smoothing
Variable singularity
M-integral
Stress intensity factor
Bi-material

ABSTRACT

This paper aims to formulate a triangular five-node (T5) singular crack-tip element in G space with strain smoothing to simulate an $r^{\lambda-1}$ $(0<\lambda<1)$ stress singularity for bi-material fracture analyses. In the present formulation, a direct point interpolation with a proper fractional order of extra basis functions is specially employed to construct variable power type singular shape functions that are in a G^1 space. Within strain smoothing, the singular terms of functions as well as mapping procedures are no longer necessary to compute the stiffness matrix. Furthermore, thanks to the point interpolation, the proposed singular element eliminates the need to shift the position of the side nodes adjacent to the crack-tip, and is thus quite straightforward and easily implemented in existing codes. The effectiveness of the present singular element is demonstrated via numerical examples of a wide range of material combinations and boundary conditions.

1. Introduction

With the increasing demands on multi-functional needs (wear, corrosion, thermal resistance and toughness) in mechanical, aerospace and biomedical applications, the development of layered material systems has come to the forefront. In the layered systems, the overall mechanical behaviors or responses hinge mainly on the cracks occurring near the interfaces between two dissimilar materials [1]. Therefore, the development of a robust and effective simulation tool to characterize the singular behavior of these cracks is crucial to a better understanding of the influence of the mismatch in properties and their effects on crack growth.

With regard to the cracks associated with the bi-material interfaces, the problem of an interface crack and that of a crack normal (and impinging) to the interface are of technical importance. It is well known that the order of singularity for the stress field is $1/\sqrt{r}$ (where r is the radial distance from the crack-tip) in typical linear elastic fracture mechanics. However, for a crack lying along a bi-material interface, the stress singularity in the vicinity of the crack-tip is oscillatory in nature, along with the presence of an inverse $\sqrt{r}$ singularity, i.e., $\sigma_{ij}\sim r^{-1/2+i\varepsilon}$ [2–4], where the oscillatory term $r^{i\varepsilon}$ depends on the mismatch in elastic properties of two bonded materials. Moreover, for the case of a crack perpendicularly terminating at the interface, Zak and Williams [5], and Cook and Erdogan [6] showed that the near-tip stress field is in the form of $\sigma_{ij}\sim r^{\lambda-1}$ $(0<\lambda<1)$, where the exponent λ is the lowest root of the characteristic equation depending on the Dundurs parameters that are related to the mechanical properties of two materials [7,8]. To model such a behavior accurately, efforts have been made to embed arbitrary order singularity $(r^{\lambda-1}, 0<\lambda<1)$ in the vicinity of the crack-tip [9–15]. Abdi and Valentin [16] generalized the idea of quarter-point elements [17] for modeling an $r^{\lambda-1}$ stress singularity, and the optimal position of side nodes adjacent to the crack-tip for quadratic and cubic isoparametric elements was obtained using a least-square method. Wu [18] extended this idea for collapsed triangular isoparametric elements and showed that they give better results. Lim and Kim [19] proposed a simpler method for calculating the optimal position of the side nodes adjacent to the crack-tip. A major disadvantage of these quadratic crack-tip elements is that the entire domain has to be, in principle, modeled by quadratic elements of the same type to ensure compatibility. Otherwise, the transition elements are needed to “bridge” the crack-tip elements to the standard elements [20]. Recently, Belytschko and Moes developed so-called extended finite element method (XFEM) to model arbitrary discontinuities

* Corresponding author at: Department of Mechanical Engineering, National University of Singapore, 9 Engineering Drive 1, Singapore 117576, Singapore. Tel.: +65 65164797.
E-mail address: g0700888@nus.edu.sg (L. Chen).

0955-7997/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.enganabound.2011.06.007