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Complex variables boundary element method for elasticity problems with constant body force

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ABSTRACT

The direct formulation of the complex variables boundary element method is generalized to allow for solving problems with constant body forces. The hypersingular integral equation for two-dimensional piecewise homogeneous medium is presented and the numerical solution is described. The technique can be used to solve a wide variety of problems in engineering. Several examples are presented to verify the approach and to demonstrate its key features. The results of calculations performed with the proposed approach are compared with available analytical and numerical benchmark solutions.

1. Introduction

Body forces of various types are important in the analysis of various engineering problems, e.g. geoengineering problems such as slope stability and tunnel design. The existing literature, and the corresponding numerical techniques, mostly deal with the analysis of gravity-induced stresses assuming that the structure is homogeneous [1–4]. The focus of the present paper is on the effects of constant body forces that may have different orientation and magnitude, e.g. gravity or seepage, on the stresses in a piece-wise homogeneous medium and on the stress intensity factors in fracture mechanics problems.

The analysis is based on the direct formulation of complex variables boundary element method for elasticity problems with constant body force. The direct boundary element formulation of elastostatics [5–7] exploits the fact that the displacements at a point within an elastic domain represented by Somigliana's displacement integral identity exactly satisfy the Navier partial differential equation, the governing equation of elasticity. By using the strain–displacement relations, Hooke's law, and traction–stress relations, Somigliana's stress and traction integral identities can be obtained. The displacements, stresses, and tractions at the boundary are determined through the limit process in which the inner point of the domain is allowed to approach its boundary.

In the case of constant body force, Somigliana's identities involve the additional integrals over the volume of the domain. Two approaches are available to avoid a volume integration (a comprehensive review is presented in [8]). The first approach, the volume integral conversion method, is to convert the domain integral to an integral over the boundary using the divergence theorem [9,10]. The second approach is to use the particular solutions [11,8], where the displacements are represented as the sum of a complementary solution that satisfies the homogeneous Navier equation (without body force) and an exact particular solution that satisfies the Navier equation with body force. The complementary solution can be represented by Somigliana's identity (without the volume integral) that involves modified boundary displacements and tractions, as described in [8]. Martel [2], Martel and Muller [3], and De Bremaecker et al. [12] used the particular solution approach in combination with the Displacement Discontinuity Method by Crouch and Starfield [6,13].

Only a few papers [14,12] address gravity effects on the elastic fields in piece-wise homogeneous medium or on the stress intensity factors at the tip of cracks. Wu and Tseng [14] used the real variables volume integral conversion method in combination with an assembly procedure that imposed the interface conditions as constraints. They used straight linear elements, and the nodal displacements and the tractions were the unknowns that were found by solving linear algebraic equations with a rather large matrix. One numerical example was discussed in that paper. De Bremaecker et al. [12] described the particular solutions approach to include gravity effects in the case of fractures. No numerical examples were discussed.

The complex variables direct boundary element method was presented in [15,16] for the case of zero body forces. The advantages of complex variables, such as easy evaluation of the integrals, use of complex arithmetic in computations, and efficient treatment of discontinuities, result in a convenient tool to treat two-dimensional problems of elasticity and fracture mechanics [15–18].

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