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A boundary integral approach for plane analysis of electrically semi-permeable planar cracks in a piezoelectric solid

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ABSTRACT

A plane electro-elastostatic problem involving arbitrarily located planar stress free cracks which are electrically semi-permeable is considered. Through the use of the numerical Green's function for impermeable cracks, the problem is formulated in terms of boundary integral equations which are solved numerically by a boundary element procedure together with a predictor–corrector method. The crack tip stress and electric displacement intensity factors can be easily computed once the boundary integral equations are properly solved.

1. Introduction

In the 1970s, Snyder and Cruse [20] pioneered the approach of using special Green's functions (modified fundamental solutions) in the boundary integral method for solving crack problems. They derived an analytical Green's function for a stress free planar crack in an infinite orthotropic elastic space and applied it to analyze the stress distribution around the crack in a body of finite extent. The work in [20] was subsequently extended by other researchers to solve more complicated crack problems (see, for example, Ang [2], Ang and Clements [3] and Clements and Haselgrove [8]). The main advantage of Green's function approach is that the singular behaviors of the stress at the crack tips are accurately captured in the boundary integral formulation of the crack problem. Furthermore, through the use of an appropriate Green's function, no integration over the crack faces is required in the boundary integral method.

In general, Green's functions for cracks with arbitrary geometries, configurations and boundary conditions are, however, difficult (if not impossible) to derive analytically. To solve a wider range of crack problems, Telles et al. [18] and Guimarães and Telles [15] proposed a numerical hypersingular integral approach for deriving the required Green's function. Such a numerical Green's function approach was also used by Ang and Telles [4] to solve an elastostatic problem involving multiple interacting planar cracks in an anisotropic body.

Recently, Athanasius et al. [5] extended the analysis in [4] to a plane electro-elastostatic crack problem, deriving numerical Green's functions for arbitrarily located stress free planar cracks which are either electrically impermeable or permeable. In the present paper, the numerical Green's function for the impermeable cracks is used to obtain boundary integral equations for multiple stress free electrically semi-permeable cracks. Because of the electrically semi-permeable conditions on the cracks, the boundary integral equations contain integrals whose integrands are given by a nonlinear function of the crack opening displacement and the electrical potential jump on the cracks. The boundary integral equations can be solved by using a simple numerical procedure if the crack opening displacement and the electric potential jump are known. Those physical quantities on the cracks are, however, not known a priori. A predictor–corrector approach which iterates to and fro estimating the crack opening displacement and the electrical potential jump and solving the boundary integral equations is presented here for the numerical solution of the semi-permeable crack problem.

A brief review of existing boundary integral approaches for solving piezoelectric crack problems may be appropriate at this juncture. Analytical closed form Green's functions which can be used to derive boundary element solutions for a single stress free planar crack which is either impermeable or conducting (permeable) are given in Rajapakse and Xu [17]. Garcia-Sanchez et al. [12] and Groh and Kuna [13] presented numerical procedures based on boundary integral equations derived by using fundamental solution which does not satisfy the boundary conditions on the crack faces. In [13], opposite crack faces were modeled by using the so-called subdomain technique and quarter-point elements were employed to deal with the singular behaviors of the stress and electric displacement at the crack tips, while a dual (mixed) boundary integral formulation was used in [12] with the conditions on the cracks treated by a differentiated form of the usual boundary

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