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A coupled boundary element-finite difference solution of the elliptic modified mild slope equation

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ABSTRACT

The modified mild slope equation of [5] is solved using a combination of the boundary element method (BEM) and the finite difference method (FDM). The exterior domain of constant depth and infinite horizontal extent is solved by a BEM using linear or quadratic elements. The interior domain with variable depth is solved by a flexible order of accuracy FDM in boundary-fitted curvilinear coordinates. The two solutions are matched along the common boundary of two methods (the BEM boundary) to ensure continuity of value and normal flux. Convergence of the individual methods is shown and the combined solution is tested against several test cases. Results for refraction and diffraction of waves from submerged bottom mounted obstacles compare well with experimental measurements and other computed results from the literature.

1. Introduction

Water wave propagation from deep to shallow water is a major concern in coastal engineering. Waves feel the bed as the domain geometry changes when they approach the shoreline. Under the assumptions of a potential flow, the governing equation for the scattering of water waves over a bed topography is Laplace's equation subject to appropriate boundary conditions. The original mild-slope equation (MSE) of [2] was derived by assuming the linear, constant depth solution to be locally valid and integrating over the depth. The result is a two-dimensional elliptic equation describing linear wave scattering over moderately varying bathymetry which captures both refraction and diffraction effects. Booij [3] showed that the mild-slope equation can give accurate results even with a plane bottom slope up to 1:3. An extension to steeper bottom slopes was given by [13] who verified the improvement by considering scattering from ripple patches. Using a Galerkin eigenfunction expansion, [18] presented an extended mild-slope equation (EMSE) which includes the effect of both evanescent and propagating modes and is capable of treating relatively steep bed profiles. The resulting equation includes higher order terms of the bottom slope and the term proportional to the bottom curvature, as well as the evanescent modes. Chamberlain and Porter [5] derived the modified mild-slope equation (MMSE) that retains second-order terms, and [22] derived a solution that takes into account the terms associated with the evanescent modes with depth averaged mass flux and pressure boundary conditions for the case of unidirectional wave transformation. Suh et al. [26] used Green's second identity and a Lagrangian formulation to develop two equivalent time-dependent equations for wave propagation over a rapidly varying topography. Chandrasekera and Cheung [6] derived an elliptic refraction–diffraction equation that includes the bottom curvature and slope-squared terms. In their numerical solutions, it was found that using the extended model to calculate cases of a rapidly varying seabed produced better results than the traditional mild-slope equation. Following the procedure outlined by [7,14] developed the extended hyperbolic mild-slope equation to account for wave transformations over a rapidly varying topography. Their results based on an FDM solution showed that the hyperbolic equation had the same accuracy as the extended elliptic equation. Hsu and Wen [11] re-cast the extended refraction–diffraction equation into a time-dependent equation. A comprehensive review of the original mild slope equation as well as its extended and modified versions may be found in [33].

The modified mild slope equation has been solved numerically using the FDM, the finite element method (FEM) and a form of the BEM called the dual reciprocity boundary element method (DRBEM). The solutions discussed above were implemented using the FDM, as are those presented by [20,24,16] among others. The DRBEM method was developed by [19] and later extended and applied to this problem by many others including [32]. More recent variations are given by [34,35]. Each of these discrete

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