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A meshless method for solving nonhomogeneous Cauchy problems ☆

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ABSTRACT

In this paper the method of fundamental solutions (MFS) and the method of particular solution (MPS) are combined as a one-stage approach to solve the Cauchy problem for Poisson's equation. The main idea is to approximate the solution of Poisson's equation using a linear combination of fundamental solutions and radial basis functions. As a result, we provide a direct and effective meshless method for solving inverse problems with inhomogeneous terms. Numerical results in 2D and 3D show that our proposed method is effective for Cauchy problems.

1. Introduction

We consider a bounded and connected domain $\Omega \subset \mathbb{R}^d$, $d=2,3$ and the following classical Cauchy problem:

$$\Delta u(\mathbf{x}) = f(\mathbf{x}), \quad \mathbf{x} \in \Omega, \tag{1}$$

with measured Dirichlet and Neumann data on the known fixed boundary Γ (accessible for data measurement), which is a portion of the boundary $\partial\Omega$

$$u(\mathbf{x}) = g(\mathbf{x}), \quad \mathbf{x} \in \Gamma, \tag{2}$$

$$\frac{\partial u(\mathbf{x})}{\partial \nu} = h(\mathbf{x}), \quad \mathbf{x} \in \Gamma, \tag{3}$$

where ν is the unit outward normal vector on Γ, and $f(\mathbf{x})$, $g(\mathbf{x})$, $h(\mathbf{x})$ are given functions.

The above-mentioned Cauchy problem is a typical ill-posed problem in the sense of Hadamard [13] that any small error in the measured data may induce enormous error to the solution. The uniqueness and conditional stability of the solution to the problem (1)–(3) was given by Bukhgeim et al. [3]. The Cauchy problem for the Laplace equation arises from many branches of science and engineering, for example, non-destructive testing [3,16,36]. Recently, many numerical methods were employed to solve such problem, for instance, Backus-Gilbert algorithm [16], FEM [8], BEM [6] and MFS [33–35]. In general, the solution of the inverse problem does not depend continuously on the initial data, as shown by Wei and Hon [32] for a Cauchy problem which often arises in monitoring the possibility of boundary corrosion in iron melting process.

During the past decades, the method of fundamental solution (MFS) is considered as a meshless method and has been proven to be a highly effective boundary meshless method when the fundamental solutions of the governing equations are available [9,12]. Compared with the traditional mesh methods (FDM, FEM and BEM) [10], the meshless methods have the following advantages:

- It is applicable to more complicated domains.
- It is readily extendable to solve high-dimensional problems.
- It can be extended to solve time-dependent problems with known fundamental solutions.

The MFS was first introduced by Kupradze and Aleksidze [22] in 1964. It had been largely applied to solve various types of homogeneous partial differential PDEs [9,12]. For instances, the solutions for potential problems by Mathon and Johnston [26], exterior Dirichlet acoustic scattering problem by Kress and Mohsen [20], and general second order linear elliptic partial differential equations by Clements [7]. Recently, Hon and Wei applied the MFS to solve the Cauchy problem of Laplace equation and heat equation in one-dimension [17], multidimensions [19], and for various kinds of boundary conditions [18]. The MFS also

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