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A comparison between a symmetric and a non-symmetric Galerkin finite element—boundary integral equation coupling for the two-dimensional exterior Stokes problem

A.J. Radcliffe

Institut für Angewandte Mathematik, Gottfried Wilhelm Leibniz Universität Hannover, Am Welfengarten 1, 30167 Hannover, Germany

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ABSTRACT

Symmetric and non-symmetric Galerkin formulations are presented for the coupling of a finite element modelled interior region to a boundary integral supported exterior region for the two-dimensional steady state exterior Stokes problem. Both single and double-layer hydrodynamic potentials are used allowing a well conditioned symmetric matrix structure for the entire interior–exterior, velocity–pressure system when the exterior velocity boundary integral equation (VBIE) is augmented by a traction boundary integral equation (TBIE) with the pressure determined everywhere purely through the imposition of the divergence-free velocity condition. Corresponding non-symmetric formulations are obtained by additionally discretizing an associated pressure boundary integral equation (PBIE), where the associated kernel functions have singularities an order higher than in the VBIE, with a simple regularization of the new hyper-singular pressure kernel. Comparable solution convergence with mesh refinement for the symmetric and non-symmetric schemes is shown for stabilized and mixed velocity–pressure conforming finite element pairs using Lagrangian shape functions.

1. Introduction

For open problems involving non-linearities, where solutions are required over domains of infinite extent but perhaps exhibit a non-linear (or just very complex) behaviour local to the origin, there has been a significant amount of work coupling the finite-element method (FEM) to the boundary element method (BEM).

While a true non-linear solution behaviour, typically restricted to an ‘‘interior’’ region, is not amenable to a boundary element treatment without significant simplification, by only requiring the boundaries of computational domains to be discretized, boundary elements have proved invaluable as a form of interior region mesh termination; the decaying influence towards infinity of disturbances emanating from within the (finite element, say) supported interior about the origin being captured accurately by the very nature of the boundary element single/double-layer potentials that radiate such information away from sources located along the interior's interface with the ‘‘exterior’’ region.

Of course, the price to pay for such a dimensional reduction of the entire exterior region to use a discretization just on its boundary is the introduction of singularities when the boundary potentials are required to evaluate fields very close to, or rather *on*, their boundary sources to generate the BEM matrices; and it is the question of whether it is worth introducing and regularizing the singularities associated with a further particular boundary integral equation, that for the (exterior) pressure, that will be the central pre-occupation of the present work.

The earliest FEM–BEM coupling approaches used just a single integral equation – equivalent to the velocity boundary integral equation (VBIE) here – and either the single-layer or the double-layer (for better conditioning) potential operators [1–4]. Such schemes continued to be used for Stokes type problems [5,6] even after they were joined by completely symmetric formulations, initiated by [7], especially suited to linear elastic (and thus Stokes) systems.

These symmetric methods additionally took a further derivative of the original boundary integral equation – to give the equivalent of the traction boundary integral equation (TBIE) here – and thus used two integral equations to support the exterior region [7–10]. This had the drawback of introducing a hyper-singular operator when double-layer potentials were used. However, for the linear elastic case (upon which the present Stokes analysis is based) this operator was later neatly regularized by a simple double integration by parts [11], the method that will be used here.

The derivation of the symmetric Galerkin equations for the Stokes problem that will be briefly presented for completeness, closely parallels that performed for incompressible linear elasticity in [12], using the symmetric coupling of FEM with BEM as

E-mail addresses: radcliffe@ganil.fr, radcliffe@ifam.uni-hannover.de

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