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Optimality of the method of fundamental solutions

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ABSTRACT

The Effective-Condition-Number (ECN) is a sensitivity measure for a linear system; it differs from the traditional condition-number in the sense that the ECN is also right-hand side vector dependent. The first part of this work, in [EABE 33(5): 637-43], revealed the close connection between the ECN and the accuracy of the Method of Fundamental Solutions (MFS) for each given problem. In this paper, we show how the ECN can help achieve the problem-dependent quasi-optimal settings for MFS calculations—that is, determining the position and density of the source points. A series of examples on Dirichlet and mixed boundary conditions shows the reliability of the proposed scheme; whenever the MFS fails, the corresponding value of the ECN strongly indicates to the user to switch to other numerical methods.

1. Introduction

The Method of Fundamental Solutions (MFS) is a popular numerical method for solving homogeneous boundary value problems. For simplicity, our presentation is restricted to the homogeneous Poisson problem

$$\Delta u = 0 \quad \text{in } \Omega \subset \mathbb{R}^2,$$

$$\partial_n^{(k)} u = f_k \quad \text{on } \Gamma_k \subset \partial\Omega,\ k \in \{0,1\}, \tag{1}$$

where the operator ∂_n is the outward-normal derivative, $\Gamma_0 \cup \Gamma_1 = \partial\Omega$, $\Gamma_0 \cap \Gamma_1 = \emptyset \neq \Gamma_0$, and in this paper, the functions f_0 and f_1 are called the *boundary data functions* which are used to generate boundary data. The MFS, belonging to a special class of Trefftz Methods [17,18], approximates the solution of the boundary value problem (1) by linear combinations of fundamental solutions centered at source-points located outside the domain of interest. Unknown coefficients are sought to best-fit the boundary data with the singularities not ever going into the domain Ω; this is done usually by collocation but other weak-formulations work too. The applications of the MFS are very wide: from linear problems [7,15,31], to nonlinear equations [1,3], and to inverse problems [4,30]. Thorough surveys on the MFS can be found in [6,9].

Recent research on the MFS is extensive and it is commonly believed that the MFS can always achieve highly accurate solution up to the order of machine precision. Recently, Schaback [27] made the following observation. It is usually due to convenience that many researchers choose harmonic functions to be boundary data functions for verifying the accuracy of the MFS; that is, both $\Delta f_0 = 0$ and $f_1 = \partial_n f_0$ hold in $\mathbb{R}^2$. With these *globally harmonic* boundary data functions, the MFS calculations are always stable and its results are always accurate; both facts hold independently of the shape of Ω. Moreover, using harmonic polynomial approximations will do even better than the MFS in such situations. Many applications in engineering and science give rise to boundary data functions which are "not-that-nice". For example, the boundary control method in [23] gives subproblems with f_0 being fundamental solutions but $f_1 \equiv 0$. This either means the solution to (1) has a finite harmonic-extension outside the domain Ω or, in the serious situations, the solution or one of its derivatives has a singularity on the boundary $\partial\Omega$. All these *facts* do not make the MFS impractical, but one needs to be more cautious when employing the method. The solution provided in [27] is an adaptive algorithm that selects an appropriate basis (either a fundamental solution or a harmonic polynomial) iteratively. The algorithm there is one variation of the greedy algorithms for asymmetric meshless collocation methods [12,14,22,20,21] and it shares some common features to the matching-pursuit algorithm [25] for image processing. The full details are omitted here and we are going to present another alternative from a very different approach.

2. MFS linear systems and ECN

Let $\tilde{\Omega} \supset \Omega$ be the fictitious domain. The set-up of the MFS linear system often involves placing a set of M collocation points $X = \{x_1, \ldots, x_M\}$ on the domain boundary and a set of N source points $\Xi = \{\xi_1, \ldots, \xi_N\}$ on the fictitious boundary $\partial\tilde{\Omega}$. The MFS

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