



Dynamic element discretization method for solving 2D traction boundary integral equations

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ABSTRACT

A sufficient condition for the existence of element singular integral of the traction boundary integral equation for elastic problems requires that the tangential derivatives of the boundary displacements are Hölder continuous at collocation points. This condition is violated if a collocation point is at the junction between two standard conforming boundary elements even if the field variables themselves are Hölder continuous there. Various methods are proposed to overcome this difficulty. Some of them are rather complicated and others are too different from the conventional boundary element method. A dynamic element discretization method to overcome this difficulty is proposed in this work. This method is novel and very simple: the form of the standard traction boundary integral equation remains the same; the standard conforming isoparametric elements are still used and all collocation points are located in the interior of elements where the continuity requirements are satisfied. For boundary elements with boundary points where the field variables themselves are singular, such as crack tips, corners and other boundary points where the stress tensors are not unique, a general procedure to construct special elements has been developed in this paper. Highly accurate numerical results for various typical examples have been obtained.

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1. Introduction

Two-dimensional elastic boundary-value problems with various kinds of singular boundary conditions will be investigated by boundary integral equations (BIEs) using boundary element methods (BEMs) in this paper. The BIE formulations and the shape functions used in BEM may produce great effect on the accuracy of the numerical computations. For simplicity, let u_i , u'_i and t_i designate boundary displacements, tangential derivatives of boundary displacements and boundary tractions of an elastic body, respectively. It is well known that u_i is continuous for all boundary points. However, u'_i and t_i at some boundary points, which are called singular boundary points in this paper, may have various types of singularities, such as $\ln r$ weak singularity, discontinuity of the first kind and singularity of $r^{-\lambda}$ ($0 < \lambda < 1$). If a smooth boundary point is singular, the stress tensor at that point is multi-valued and/or un-bounded. The situation of a corner, on the other hand, is more complex. Since u'_i and t_i of corners are generally discontinuous, all corners are singular boundary points. However, the stress tensor at a corner may be unique or it may be multi-valued and/or un-bounded. If stress

tensors are unique for all boundary points, very good BEM numerical results can be obtained using the displacement-BIE with isoparametric elements. On the other hand, traction-BIE is more suitable to treat problems with singular boundary points, in some cases (e.g., crack problems) where the use of traction-BIE appeared unavoidable.

The integral kernel related to u_i of a displacement-BIE has Cauchy singularity. It is well known that a sufficient condition for the existence of Cauchy principal-value integral is that u_i is $C^{0,\alpha}$ continuous. This condition is always satisfied for elasticity problems. The integral kernel related to u'_i of a traction-BIE has Hadamard hyper-singularity, a sufficient condition for the existence of Hadamard finite-part integral is that u_i is $C^{1,\alpha}$ continuous [1,2]. The degenerate traction-BIE with u'_i and t_i as field variables will be used in this paper. Both integral kernels related to u'_i and t_i of the degenerate traction-BIE have Cauchy singularity, the sufficient condition for the existence of the singular integrals requires $C^{0,\alpha}$ continuity for u'_i and t_i . Therefore, the sufficient condition for the existence of the singular integrals still requires $C^{1,\alpha}$ continuity for u_i . A weaker condition for the existence of singular integrals of various kinds of BIEs has been provided in [2].

In traditional BEM, piecewise continuous shape functions composed of the Lagrangian polynomials are often used to approximately describe the geometry of elements where the

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