Engineering Analysis with Boundary Elements 35 (2011) 342–354

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

The MFS for numerical boundary identification in two-dimensional harmonic problems

Liviu Marin [a,*], Andreas Karageorghis [b], Daniel Lesnic [c]

[a] *Institute of Solid Mechanics, Romanian Academy, 15 Constantin Mille, P.O. Box 1-863, 010141 Bucharest, Romania*
[b] *Department of Mathematics and Statistics, University of Cyprus/Πανεπιστήμιο Κύπρου, P.O. Box 20537, 1678 Nicosia/Λευκωσία, Cyprus/Κύπρος*
[c] *Department of Applied Mathematics, University of Leeds, Leeds LS2 9JT, UK*

ARTICLE INFO

Article history:
Received 16 June 2010
Accepted 26 September 2010
Available online 20 October 2010

Keywords:
Inverse problem
Method of fundamental solutions (MFS)
Regularization

ABSTRACT

In this study, we briefly review the applications of the method of fundamental solutions to inverse problems over the last decade. Subsequently, we consider the inverse geometric problem of identifying an unknown part of the boundary of a domain in which the Laplace equation is satisfied. Additional Cauchy data are provided on the known part of the boundary. The method of fundamental solutions is employed in conjunction with regularization in order to obtain a stable solution. Numerical results are presented and discussed.

1. Introduction

The method of fundamental solutions (MFS) is a meshless boundary collocation method which is applicable to boundary value problems in which a fundamental solution of the operator in the governing equation is known explicitly. Despite this restriction, it has, in recent years, become very popular primarily because of the ease with which it can be implemented, in particular for problems in complex geometries. The basic ideas of the method were first introduced by Kupradze and Aleksidze in the early 1960s, see e.g. [31]. Since its introduction as a numerical method by Mathon and Johnston [44], it has been successfully applied to a large variety of physical problems, an account of which may be found in the survey papers by Fairweather and Karageorghis [13], Fairweather et al. [14] and Golberg and Chen [17].

The ease of implementation of the MFS for problems with complex boundaries makes it an ideal candidate for problems in which the boundary is of major importance or requires special attention, such as free boundary problems. A different but related class of problems to which the MFS is naturally suited is the class of inverse problems. Inverse problems can be subdivided into four main categories, namely Cauchy problems, inverse geometric problems, source identification problems and parameter identification problems. For these reasons, the MFS has been used increasingly over the last decade for the numerical solution of the above classes of problems.

The aim of this paper is, after briefly surveying the applications of the MFS to inverse problems in recent years, to study the application of the method to inverse geometric problems for the Laplace equation subject to various boundary conditions, and to present the various implementational issues related to this application. More specifically, we shall consider the inverse boundary value problem given by the Laplace equation

$$\Delta u = 0 \quad \text{in } \Omega, \tag{1a}$$

subject to the boundary conditions

$$u = f_1 \quad \text{and} \quad \frac{\partial u}{\partial \boldsymbol{\nu}} = g_1 \quad \text{on } \partial\Omega_1, \tag{1b}$$

$$u = f_2 \quad \text{on } \partial\Omega_2, \tag{1c}$$

or

$$\frac{\partial u}{\partial \boldsymbol{\nu}} = g_2 \quad \text{on } \partial\Omega_2, \tag{1d}$$

where $\Omega \subset \mathbb{R}^d$ is a bounded domain, d is the dimension of the space where the problem is posed, usually $d \in \{1,2,3\}$, f_1, g_1, f_2 and g_2 are known functions and the boundary $\partial\Omega = \partial\Omega_1 \bigcup \partial\Omega_2$, where $\partial\Omega_1$ is the known part of the boundary and $\partial\Omega_2$ is the unknown part of the boundary to be identified. Also, $\partial/\partial\boldsymbol{\nu}$ denotes the partial derivative in the direction of the outward unit normal vector $\boldsymbol{\nu}$ to the boundary. In Eqs. (1b) and (1c), $\partial\Omega_1$ and $\partial\Omega_2$ are, in general, two simple arcs having in common the endpoints only, and this problem occurs in several contexts such as corrosion detection by electrostatic measurements of the voltage f_1 and the current flux g_1, see Kaup and Santosa [30], or crack detection in non-ferrous metals subject to electromagnetic measurements, see McIver [45].

* Corresponding author.
E-mail addresses: marin.liviu@gmail.com, liviu@imsar.bu.edu.ro (L. Marin), andreask@ucy.ac.cy (A. Karageorghis), amt5ld@maths.leeds.ac.uk (D. Lesnic).

0955-7997/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.enganabound.2010.09.014