Engineering Analysis with Boundary Elements 35 (2011) 68–76

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Radial basis function approximation methods with extended precision floating point arithmetic

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ARTICLE INFO

Article history:
Received 12 October 2009
Accepted 26 May 2010
Available online 17 June 2010

Keywords:
RBF interpolation
RBF collocation for PDEs
Extended precision floating point arithmetic

ABSTRACT

Radial basis function (RBF) methods that employ infinitely differentiable basis functions featuring a shape parameter are theoretically spectrally accurate methods for scattered data interpolation and for solving partial differential equations. It is also theoretically known that RBF methods are most accurate when the linear systems associated with the methods are extremely ill-conditioned. This often prevents the RBF methods from realizing spectral accuracy in applications. In this work we examine how extended precision floating point arithmetic can be used to improve the accuracy of RBF methods in an efficient manner. RBF methods using extended precision are compared to algorithms that evaluate RBF methods by bypassing the solution of the ill-conditioned linear systems.

1. Introduction

IEEE 64-bit floating-point arithmetic (double precision) is sufficiently accurate for most scientific applications. However, for a rapidly growing body of important scientific computing applications, a higher level of numeric precision is required. These applications include supernova simulations, climate modeling, planetary orbit calculations, Coulomb n-body atomic systems, scattering amplitudes of quarks, gluons and bosons, nonlinear oscillator theory, quantum field theory, and experimental mathematics [1,2]. In this work, we show that radial basis function (RBF) approximation methods are another area that benefit from extended numerical precision.

In Ref. [3], numerical experiments with Elliptic PDEs were performed using *Mathematica*'s arbitrary precision package with 100-digit accuracy to gain some insight into the connection between the accuracy of the RBF method using the inverse multiquadric RBF, maximum grid spacing, and the shape parameter. The authors determined that for a given grid spacing, an optimal value of the shape parameter exists whose value should not be decreased unless the grid spacing was refined. Also in [3], the authors concluded that in order to achieve optimal accuracy and efficiency in solving elliptic boundary value problems, it is better to use a relatively coarse grid and extended precision than standard precision and a fine grid. The authors in [3] present an extended precision calculation using a software package written in the C++ programming language that is available at [4]. They conclude that while promising, that since extended precision computations with C++ is relatively new, further investigation in this direction is needed. In this work we continue to investigate whether extended precision calculations using C++ can improve the accuracy and efficiency of RBF methods.

2. Radial basis function approximation

In this section the RBF interpolation method and the RBF asymmetric collocation method (Kansa's method) [5] for the solution of steady PDEs are summarized. Over the last 25 years, RBF methods have become an important tool for the interpolation of scattered data and for solving partial differential equations [6]. Recent books [7,8,6,9] on RBF methods can be consulted for more information.

The RBF interpolation method uses linear combinations of translates of one function $\phi(r)$ of a single real variable. Given a set of *centers* $\mathbf{x}_1^c,\ldots,\mathbf{x}_N^c$ in $\mathbb{R}^d$, the RBF interpolant takes the form

$$s(\mathbf{x}) = \sum_{j=1}^{N} \alpha_j \phi(\|\mathbf{x}-\mathbf{x}_j^c\|_2,\varepsilon), \tag{1}$$

where

$$r = \|\mathbf{x}\|_2 = \sqrt{x_1^2+\cdots+x_d^2}.$$

We focus on RBFs $\phi(r)$ that are infinitely differentiable and that contain a free parameter, ε, called the shape parameter. Some examples from this class of RBF are listed in Table 1. In all the numerical examples, we have used the MQ which is representative of this class and is popular in applications. The coefficients, α, are chosen by enforcing the interpolation

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doi:10.1016/j.enganabound.2010.05.011