Engineering Analysis with Boundary Elements 35 (2011) 77–84

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A recursive application of the integral equation in the boundary element method

C.F. Loeffler *

Mechanical Engineering Department, Federal University of Espírito Santo, Av. Fernando Ferrari, 514 Vitória, ES – CEP 29075-910, Brazil

ARTICLE INFO

Article history:
Received 5 August 2009
Accepted 26 May 2010
Available online 19 June 2010

Keywords:
Boundary element method
Method of weighted residuals
Scalar potential problems

ABSTRACT

This paper presents a recursive application of the governing integral equation aimed at improving the accuracy of numerical results of the boundary element method (BEM). Usually, only the results at internal domain points when using BEM are found using this approach, since the nodal boundary values have already been calculated. Here, it is shown that the same idea can be used to obtain better accuracy for the boundary results as well. Instead of locating the new source points inside the domain, they are positioned on the boundary, with different coordinates to the nodal points. The procedure is certainly general, but will be presented using as an example the two dimensional Laplace equation, for the sake of simplicity to point out the main concepts and numerical aspects of the method proposed, especially due to the determination of directional derivatives of the primal variable, which is part in hyper-singular BEM theory.

1. Introduction

Due to the mathematical modeling advances, improved computational capability and data storage, some traditional numerical techniques have been subjected to adaptations in order to achieve better performance. After procedures such as the relocation of discretization points, adaptive mesh refinement and adaptive shape functions, in some numerical methods an iterative solution scheme has been introduced in order to improve the numerical accuracy. Following this tendency, the recursive use of the boundary integral equation is here presented as an auxiliary resource to improve boundary element method (BEM) performance. It is not a completely iterative technique, but a simple procedure based on a common scheme to calculate internal values with BEM, where the boundary integral equation is used twice.

2. Laplace boundary integral equation

Considering an auxiliary functions $u^*(\xi;X)$ and its normal derivative $q^*(\xi;X)$, where ξ is an internal domain point, and considering the basic mathematical tools of the theory of integral equations, the boundary integral equation associated to the Laplace Equation can be written [1] as

$$u(\xi)=\int_{\Gamma}u^*(\xi;X)q(X)\,\mathrm{d}\Gamma-\int_{\Gamma}q^*(\xi;X)u(X)\,\mathrm{d}\Gamma \tag{1}$$

In Eq. (1), $q(X)$ is the potential normal derivative. The essential ($u=\overline{u}$) and natural ($u_{,i}n_i=\overline{q}$) boundary conditions are defined on the boundary $\Gamma=\Gamma_u+\Gamma_q$. The external unit normal vector at point $X=X(x_i)$ is n_i.

For the usual direct BEM formulation, the auxiliary functions concern the solution of the related problem governed by a Poisson Equation [2], for which a unit singular source exists at $X=\xi$ for an infinite domain $\Omega^*(X)$.

When the source point is positioned on the boundary, the integral equation for source points located on the boundary is given by

$$c(\xi)u(\xi)=\int_{\Gamma}u^*(\xi;X)q(X)\,\mathrm{d}\Gamma-CPV\int_{\Gamma}q^*(\xi;X)u(X)\,\mathrm{d}\Gamma \tag{2}$$

An ingenious mathematical procedure defines the $c(\xi)$ value, which is equal to 0.5 for smooth boundaries [3]. When the source point is positioned on the boundary, $u^*(\xi;X)$ and $q^*(\xi;X)$ functions present singularities. In spite of being discontinuous, $u^*(\xi;X)$ is integrable along the boundary, but the integral of $q^*(\xi;X)$ exists in the Cauchy principal value (CPV) sense.

According to the well-known BEM procedure [4], the boundary integral is discretized, generating a system of equations given in matrix notation as

$$H\Theta-GQ=0 \tag{3}$$

* Tel.: +55 27 33352669.
E-mail address: carlosloeffler@bol.com.br

0955-7997/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.enganabound.2010.05.012