Engineering Analysis with Boundary Elements 35 (2011) 543–549

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Development of circular arc boundary elements method

Mehdi Dehghan *, Hossein Hosseinzadeh

Department of Applied Mathematics, Faculty of Mathematics and Computer Science, Amirkabir University of Technology, No. 424, Hafez Ave., 15914 Tehran, Iran

ARTICLE INFO

Article history:
Received 26 November 2009
Accepted 13 August 2010
Available online 16 September 2010

Keywords:
Boundary elements method (BEM)
2D Laplace equation
Discretization error
Circular arc elements (CAE)
Error analysis

ABSTRACT

The boundary element method (BEM) is a popular technique to solve engineering problems. We compare the circular arc elements (CAE) discretization to both linear and quadratic discretizations. The main aim of this paper is to determine analytical expressions for the discretization error in 2D BEM for the Laplace equation using CAE discretization. The results are validated by numerical examples.

1. Introduction

The boundary element method (BEM) [1,2] has been used successfully to solve problems in engineering. The core task of the BEM is to determine unknown surface sources on boundaries. The accuracy of the BEM depends significantly on the order of the approximation [3]. Piecewise-constant approximations to surface sources lead to discontinuities in the approximation of these sources and a subsequent reduction in the accuracy of the computation [3,4]. To overcome the reduction in accuracy of piecewise-constant approximations it has been proposed that isoparametric boundary elements be used to approximate unknown surface sources on the boundary [4]. A drawback of using isoparametric boundary elements is the large number of nodes required for computational accuracy. To determine the number of required computational nodes the order of the interpolating polynomial must be multiplied by the number of points used to discretize the boundary [4,6]. An improvement on the use of interpolating polynomials is to use spline functions [5]. The discretization errors can be ignored by assuming the boundary is a polygon [7–9].

In this paper we investigate discretization errors that occur in the circular arc element (CAE) method. The CAE method is a boundary discretization method used in BEM. A brief discussion about CAE is outlined in Chapter 3 of [1] for the Laplace equation. In [1] the author utilized CAE to solve a 2D Laplace equation. The author did not include a discussion on the discretization errors that occur as a result of implementing the CAE. In this paper we consider the discretization errors of the CAE. We compare these with known linear and quadratic interpolations used when applying the BEM.

2. Background of BEM

Consider the following Laplace equation governing the potential problem in a $2D$ domain Ω,

$$\nabla^2 u(x) = 0,$$

under the boundary conditions : $u(x) = \overline{u(x)}$ for all $x \in S_1$, and $q(x) = \partial u/\partial n(x) = \overline{q(x)}$ for all $x \in S_2$, such that u is the potential field in domain $\Omega, \Gamma = S_1 \cup S_2$ is the boundary of Ω and n the outward normal vector. The solution of the given boundary value problem for all $x \in \Omega$ is given by (see [1,2])

$$u(x) = \oint_\Gamma [G(x,y)q(y) - H(x,y)u(y)]\, d\Gamma(y),$$

such that

$$G(x,y) = \frac{1}{2\pi}\ln(r) \quad \text{and} \quad H(x,y) = \frac{\partial G(x,y)}{\partial n(y)} = \frac{1}{2\pi r}\frac{\partial r}{\partial n}, \tag{2.1}$$

with r being the distance between the collocation point x and the field point y (Fig. 1). If $x \in \Gamma$ and boundary Γ is smooth at the collocation point x then the second Green's identity may be written as

$$\frac{1}{2}u(x) = \oint_\Gamma [G(x,y)q(x) - H(x,y)u(x)]\, d\Gamma(y).$$

The boundary Γ will be discretized (see [1]) and the values of u and q will be interpolated by polynomials. So after numerical implementation we obtain the linear equation $G\overline{q} - H\overline{u} = \overline{0}$, where

* Corresponding author.
E-mail addresses: mdehghan@aut.ac.ir, mdehghan.aut@gmail.com (M. Dehghan), h_hosseinzadeh@aut.ac.ir (H. Hosseinzadeh).

0955-7997/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.enganabound.2010.08.014