



# An analysis for the elasto-plastic problem of the moderately thick plate using the meshless local Petrov–Galerkin method

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## ABSTRACT

A meshless local Petrov–Galerkin method for the analysis of the elasto-plastic problem of the moderately thick plate is presented. The discretized system equations of the moderately thick plate are obtained using a locally weighted residual method. It uses a radial basis function (RBF) coupled with a polynomial basis function as a trial function, and uses the quartic spline function as a test function of the weighted residual method. The shape functions have the Kronecker delta function properties, and no additional treatment to impose essential boundary conditions. The present method is a true meshless method as it does not need any grids, and all integrals can be easily evaluated over regularly shaped domains and their boundaries. An incremental Newton–Raphson iterative algorithm is employed to solve the nonlinear discretized system equation. Numerical results show that the present method possesses not only feasibility and validity but also rapid convergence for the elasto-plastic problem of the moderately thick plate.

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## 1. Introduction

Atluri and Zhu [1] originally proposed a meshless Petrov–Galerkin method (MLPG), which is one of the meshless methods. MLPG method has many advantages over traditional numerical methods such as FEM and BEM. It has attracted much attention in the past decade due to its flexibility, and absolutely no elements or cells are needed in the formulation, either for interpolation purposes or for integration purposes. It requires only nodal information and no element connectivity is needed, which leads to a simple and convenient preprocessing. It is a truly meshless method. Remarkable successes of the MLPG method have been reported in solving plate bending problems [2,3]; analyses of shell deformations [4]; large deformation problems [5]; and dynamics problems [6,7].

Recently, radial basis functions (RBF) [8] have been used in meshless methods. The derived shape functions possess the Kronecker delta function property, which allow the essential boundary conditions to be imposed easily. Furthermore, when RBF is used in the local (compactly supported) rather than the global interpolation scheme, such as in MLPG method, the dense system matrices associated with the global interpolation scheme are avoided. This MLPG method is based on a weak form

computed over a local sub-domain and it is also a truly meshless method. A MLPG method for the free vibration analysis of two-dimensional solids and structures has been presented [8]; the local weak forms were developed using a local weighted residual method from the partial differential equation governing free vibrations, and using radial functions as the basis to obtain the shape functions. Liu et al. [9] found that the linear function would be reproduced when the linear polynomial is added to the basis function of the interpolation; this made the method with the linear polynomial basis much more accurate than that without the polynomial basis.

Moderately thick plates are widely used in a variety of engineering structures. Transverse shear effects must be considered in order to obtain good results. Sun et al. [10] have solved the free vibration of moderately thick plates using the element-free Galerkin method. Xiao et al. [11] have analyzed thick plates using MLPG method with radial basis functions. Xia et al. [12,13] have analyzed elastic dynamic problems of the moderately thick plate and the static bending and the free vibration problems of non-homogeneous moderately thick plates using the meshless LRPIM. In this paper, the meshless local Petrov–Galerkin method is used to solve the elasto-plastic problem of the moderately thick plate.

## 2. Fundamental equations of the moderately thick plate

A Cartesian coordinate system, shown in Fig. 1, is used to describe infinitesimal deformations of a moderately thick plate. The mid-plane of the plate is denoted by  $x$ – $y$  plane, and

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