



# The quasi-linear method of fundamental solution applied to transient non-linear Poisson problems

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## ABSTRACT

This paper proposes the use of a quasi-linear method of fundamental solution (QMFS) and explicit Euler method to treat the transient non-linear Poisson-type equations. The MFS, which is a fully meshless method, often deals with the linear and non-linear Poisson equations by approximating a particular solution via employing radial basis functions (RBFs). The interpolation in terms of RBFs often leads to a badly conditioned problem which demands special cares. The current work suggests a linearization scheme for the nonhomogeneous term in terms of the dependent variable and finite differencing in time resulting in Helmholtz-type equations whose fundamental solutions are available. Consequently, the particular solution is no longer needed and the MFS can be directly applied to the new linearized equation. The numerical examples illustrate the effectiveness of the presented method.

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## 1. Introduction

Transient non-linear Poisson problems are widely encountered in the modeling of physical phenomena. For example, transient heat conduction or mass diffusion with source terms arises in model equations in many different areas of computational physics and engineering. Representative prototype problems include transient diffusion with chemical reaction in a catalyst pellet, microwave heating process, spontaneous combustion, and thermal explosion problems and transient convection.

The numerical solution procedure usually depends on finite-difference, finite-element, boundary-element or spectral methods. The boundary element method (BEM) is one such method suitable for linear problems [1,2]. For transient problems, the BEM can be used in conjunction with finite differencing in time [3,4]. The resulting formulation is a steady-state type of Poisson equation that can be solved by dual reciprocity methods (DRM) [1]. Thus, the advantages of the boundary only discretization are retained, and the internal points are needed only for the interpolation of the nonhomogeneous terms. The disadvantage of all the BEM-based techniques is the need for the evaluation of singular or near-singular integral which can be time consuming.

As an alternative, solution methods based on the method of fundamental solution (MFS) are gaining considerable attention [5,6]. These methods are based on fitting of the boundary conditions with the fundamental solutions of the Laplace equation as the basis functions [7,8]. The poles or singularities

of the fundamental solutions are placed outside the domain, thus avoiding the need for evaluation of the singular integrals in contrast to traditional BEM. Similar to the BEM the MFS is in disadvantage when the fundamental solution of the underlying equation is not available. In this case a part of the equation, whose fundamental solution is provided, is considered as a homogeneous equation for which the MFS can be directly applied and the global solution is then obtained by assembling a particular solution and the homogeneous solution [9,10]. This method has been demonstrated for various linear differential equations and in conjunction with the method of particular solutions for non-linear Poisson problems [11,12]. In view of the rapid development of the MFS-RBF method in recent years, the applications to transient problems would be interesting. However, the application of the MFS-RBF method to transient problems has been limited. For linear transient problems, procedures based on finite differencing in time need to be used [13]. The simplest method is to use an explicit Euler method for approximating the time derivatives, and a paper by Golberg and Chen [14] provides a detailed computational study based on this approach. Also, the forcing function  $f$  was approximated at the previous time step in their study. The explicit scheme presented in their study is first-order accurate and has stability restrictions.

The use of RBFs usually leads to an ill-conditioned problem. There have been some approaches including preconditioning, locally supported RBFs [15] and domain decomposition [16] to treat the conditioning. In addition, the use of Hermite interpolation, called oscillatory RBF (OS-RBF) has been proposed to improve the interpolation quality [13,17].

To avoid ill-conditioning, this work proposes a quasi-linear MFS (QMFS) and explicit Euler method for approximating the time derivatives for non-linear transient Poisson problems, as a

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