



Boundary element analysis of 2D thin walled structures with high-order geometry elements using transformation

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ARTICLE INFO

Article history:

Received 27 November 2009

Accepted 13 July 2010

Available online 12 August 2010

Keywords:

Boundary element method

Nearly singular integrals

Thin bodies

Curved surface elements

Transformation

Elasticity problem

ABSTRACT

Thin structures have been widely designed and utilized in many industries. However, the analysis of the mechanical behavior of such structures represents a very challenging and attractive task to scientists and engineers because of their special geometrical shapes. The major difficulty in applying the boundary element method (BEM) to thin structures is the coinstantaneous existence of the singular and nearly singular integrals in conventional boundary integral equation (BIE). In this paper, a non-linear transformation over curved surface elements is introduced and applied to the indirect regularized boundary element method for 2-D thin structural problems. The developed transformation can remove or damp out the nearly singular properties of the integral kernels, based on the idea of diminishing the difference of the orders of magnitude or the scale of change of operational factors. For the test problems studied, very promising results are obtained when the thickness to length ratio is in the orders of $1\text{E}-01$ to $1\text{E}-06$, which is sufficient for modeling most thin structures in industrial applications.

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1. Introduction

With the advances in material science and manufacturing, more and more thin structures are frequently used for the design in various industrial applications, such as coating or multi-coating on machine components, sensors in smart materials and various thin films in electronic devices. However, the widespread experimental research in thin structure problems underlies a general lack of modeling efforts that represents a great challenge to researchers in computational mechanics.

For computational models of thin structures or thin shapes in structures, two numerical methods can be employed: the finite element method (FEM) and the boundary element method (BEM). The FEM is a successful tool for the analysis of many industrial applications. However, the FEM element count increases dramatically for thin structures due to aspect ratio limitations, and the procedure therefore requires too much preprocessing and CPU time as the thickness decreases. It is long believed that the BEM is more efficient and accurate in thin structural problems due to the boundary-only discretizations and its semi-analytical nature [1–3]. However, the conventional boundary element method (CBEM) cannot be applied readily to thin structures, because of the nearly singular integral problem.

The nearly singular integrals are not singular in the sense of mathematics. However, from the point of view of numerical integrations, these integrals cannot be calculated accurately by using the conventional numerical quadrature since the integrand oscillates very fiercely within the integration interval. Although that difficulty can be overcome by using very fine meshes, the process requires too much preprocessing and CPU time. In the past decades, tremendous effort is devoted to derive convenient integral forms or sophisticated computational techniques for calculating the nearly singular integrals. These proposed methods include, but are not limited to, interval subdivision method [4,5], special Gaussian quadrature method [6], exact integration method [7–10], and various non-linear transformation methods [11–16]. In recent studies, the above methods have been reviewed in detail by Zhang et al. [17] and Zhang and Sun [18].

Among the above methods, the transformation deserves special mention due to the wide suitability and higher accuracy. Most of previous transformations can be generalized into two categories: one is removing the nearly zero factor by using another zero factor that is usually generated by Jacobian; the other one is converting the nearly zero factor in the denominator to be part of the numerator, which profits from the idea of the reciprocal transformation for the regularization of weakly singular integrals. Numerical tests show that the transformations based on the former idea are effective for the calculation of nearly weakly singular integrals but not satisfactory for nearly strong singular or nearly hypersingular integrals. The latter transformations, based on the idea of reciprocal transformation, can convert

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