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Stability analysis via condition number and effective condition number for the first kind boundary integral equations by advanced quadrature methods, a comparison ☆

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ABSTRACT

In our previous study [Huang et al., 2008, 2009, 2010 [21,24,20]; Huang and Lu, 2004 [22,23]; Lu and Huang, 2000 [38]], we have proposed advanced (i.e., mechanical) quadrature methods (AQMs) for solving the boundary integral equations (BIEs) of the first kind. These methods have high accuracy $O(h^3)$, where $h=\max_{1\leqslant m\leqslant d}h_m$ and h_m $(m=1,\ldots,d)$ are the mesh widths of the curved edge Γ_m. The algorithms are simple and easy to carry out, because the entries of discrete matrix are explicit without any singular integrals. Although the algorithms and error analysis of AQMs are discussed in Huang et al. (2008, 2009, 2010) [21,24,20], Huang and Lu (2004) [22,23], Lu and Huang (2000) [38], there is a lack of systematic stability analysis. The first aim of this paper is to explore a new and systematic stability analysis of AQMs based on the condition number (Cond) and the effective condition number (Cond_eff) for the discrete matrix $\mathbf{K}_h$. The challenging and difficult lower bound of the minimal eigenvalue is derived in detail for the discrete matrix of AQMs for a typical BIE of the first kind. We obtain Cond$=O(h_{\min}^{-1})$ and Cond_eff$=O(h_{\min}^{-1})$, where $h_{\min}=\min_{1\leqslant m\leqslant d}h_m$, to display excellent stability. Note that Cond_eff $=O(\text{Cond})$ is greatly distinct to the case of numerical partial differential equations (PDEs) in Li et al. (2007, 2008, 2009, 2010) [26,31–37], Li and Huang (2008) [27–30], Huang and Li (2006) [19] where Cond_eff is much smaller than Cond. The second aim of this paper is to explore intrinsic characteristics of Cond_eff, and to make a comparison with numerical PDEs. Numerical experiments are carried out for three models with smooth and singularity solutions, to support the analysis made.

1. Introduction

The advanced (i.e., mechanical) quadrature methods (AQMs) are proposed in [20–24,38] for the first kind boundary integral equations (BIEs) of Laplace's equation. The AQMs provide high accuracy $O(h^3)$, accompanied with low computation complexity, where $h=\max_{1\le m\le d}h_m$ and h_m $(m=1,\ldots,d)$ are the mesh widths of the curved edge Γ_m. Note that the entries of discrete matrix $\mathbf{K}_h$ are explicit without any singular integrals. Especially, for concave polygons Ω, the solution at concave corners of $\partial\Omega$ has singularities, to heavily damage accuracy of numerical solutions. The accuracy of Galerkin methods (GMs) [44,45] is only $O(h^{1+\varepsilon})$ $(0<\varepsilon<1)$ and the accuracy of collocation methods (CMs) [47] is even lower. In contrast, the accuracy of AQMs for singularities is as high as $O(h^3)$. In fact, the quadrature method was first proposed for an integral equation with a logarithmic kernel in Christiansen [11] in 1971, called the modified quadrature method, and its analysis was given in Saranen [41], to yield only the $O(h^2)$ convergence rate. In previous study [20–24,38], we propose the new quadrature methods called the AQMs, to yield the high $O(h^3)$ convergence rate. Moreover, for AQMs, by extrapolations and splitting extrapolations methods (SEMs), the higher precision of numerical solutions and a posteriori error estimates can be achieved.

This paper is devoted to its stability analysis. Consider the linear algebraic equations:

$$\mathbf{K}_h\mathbf{x}=\mathbf{b}, \tag{1.1}$$

resulting from the first kind BIE, where the $\mathbf{x}\in R^n$ and $\mathbf{b}\in R^n$ are the unknown and known vectors, respectively. The condition number

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