



Fourier differential quadrature method for irregular thin plate bending problems on Winkler foundation

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ABSTRACT

This paper describes Fourier differential quadrature method (FDQM). It is the combination of the Fourier spectral method and differential quadrature method (DQM) in barycentric form as a numerical method for solving problems for thin plates resting on Winkler foundations with irregular domains. The solution is decomposed into a polynomial particular solution for the inhomogeneous equation and the general solution for the homogeneous equation. In the solution procedure, the arbitrary distributed loading is first approximated by the Chebyshev polynomials and thus, the desired polynomial particular solution is obtained. For the latter, we use Fourier series expansion and determine the Fourier coefficients from the boundary conditions. Furthermore, the complex boundary conditions on irregular domains can be solved with DQM directly. Finally, numerical experiments are carried out to demonstrate the flexibility, high efficiency and accuracy of our method for irregular domains.

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1. Introduction

Linear elastic bending problems of plates involving complex geometries, loading and boundary conditions have been extensively studied. Besides the traditional methods, such as the finite difference method and the finite element method, the boundary element method (BEM) also has been widely used for solving these problems [1]. It should be pointed out that a lot of computational effort is required for the complicated numerical domain integration using the traditional BEM formulation. Moreover, the fundamental solution is a spatially oscillating function, and a very fine discretization mesh is desired in the case of large values of the wave number. The most important shortcoming of such a formulation is the fact that the fundamental solution is dependent on the wave number and all the integrals are to be recalculated, if the boundary-value problem is to be solved for different values of the frequency. These calculations are very time-consuming and inefficient. In the recent years, researchers have paid attention to the meshless numerical methods without employing the concept of elements. The meshfree boundary-type collocation methods have attracted a lot of attention in the numerical solutions of various partial differential equations, e.g. the method of fundamental solution (MFS) which can solve the homogeneous problems with boundary-only discretization. However, it requires inner nodes in conjunction with the other

techniques to handle inhomogeneous problems. Since 1980s, the dual reciprocity method (DRM) and the multiple reciprocity method (MRM) have emerged as the two most promising techniques to solve inhomogeneous problems [2]. In 1992, Sladek et al. proposed the MRM-BEM [3]. It could transform the domain integrals into boundary integrals which are frequency-independent. Hence, once evaluated, boundary integrals can be reemployed in computations for any frequency. Later, Golberg et al. combined the MFS and the DRM as a mature meshless numerical method and extended the MFS-DRM to Helmholtz and diffusion problems [4]. In these works, the inhomogeneous source terms were approximated by augmented polynomial spline. Furthermore, Fu et al. developed the MRM-based meshfree boundary particle method (BPM) [5]. Comparing with the MFS-DRM, since the striking advantage of MRM over the DRM is that it does not require using inner nodes at all for the particular solution, BPM requires much less computational effort. However, the MRM is computationally expensive in the construction of the interpolation matrix and has limited feasibility for general inhomogeneous problems due to its use of high-order Laplacian operators.

In this article, we are interested in thin plate bending problems on Winkler foundation with arbitrary shapes and complex boundary conditions. We present a new method based on Fourier differential quadrature method (FDQM). It combines Fourier spectral method [6] and differential quadrature method (DQM) [7]. Fourier spectral method has the advantage of high accuracy with less unknowns, but it is limited to solving problems on circle domain or with periodic initial or boundary conditions. For the traditional DQM, it is efficient and keeps high accuracy for rectangle domain. Generally, the number of the unknowns is N^2 .

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