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Regularized meshless method for nonhomogeneous problems

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ABSTRACT

The regularized meshless method is a novel boundary-type meshless method but by now has largely been confined to homogeneous problems. In this paper, we apply the regularized meshless method to the nonhomogeneous problems in conjunction with the dual reciprocity technique in the evaluation of the particular solution. Numerical experiments of three benchmark nonhomogeneous problems demonstrate the accuracy and efficiency of the present strategy.

1. Introduction

In recent years, the boundary-type meshless methods have attracted a growing number of mathematicians and engineers [1–6]. These methods require neither domain nor boundary mesh discretization which costs much time in dealing with large-scale problems in using traditional numerical methods, such as the finite difference, finite element, and boundary element methods. The method of fundamental solutions (MFS) [7,8], the boundary knot method (BKM) [9–11], and the regularized meshless method (RMM) [12–14] are typical boundary-type meshless methods.

For the above-mentioned three methods, the RMM is found to outperform than the MFS in that the fictitious boundary in the latter is completely eliminated. Especially, the full coefficient matrix of the RMM discretization equation is well-conditioned which is superior to the ones of the MFS and BKM [15]. To the best of our knowledge, the RMM [16–18] has by now been employed to solve homogeneous problems but no report is available for its solution of nonhomogeneous problems in the literature.

In this study, we employ the regularized meshless method in conjunction with the dual reciprocity method (DRM) [19] to solve nonhomogeneous problems. This strategy can be viewed as a two-step method. First, the DRM is used to approximate the particular solution, and then the RMM is employed to calculate the homogeneous solution. In order to show the accuracy and validity, the proposed approach is tested to three benchmark examples.

The rest of the paper is divided into four sections. In Section 2, we briefly introduce the dual reciprocity method to evaluate the particular solutions, followed by Section 3 where we review the basic methodology of the regularized meshless method. Three numerical examples are investigated in Section 4. Section 5 draws some concluding remarks.

2. Dual reciprocity method for the particular solution

Without loss of generality, we consider the following boundary value problem:

$$\nabla^2 u(\mathbf{x}) = f(\mathbf{x}) \quad \text{in } \Omega \tag{1}$$

$$u(\mathbf{x}) = g(\mathbf{x}) \quad \text{on } \Gamma_D \tag{2}$$

$$\frac{\partial u(\mathbf{x})}{\partial \vec{n}} = h(\mathbf{x}) \quad \text{on } \Gamma_N \tag{3}$$

where ∇^2 is the Laplace operator and $f(x)$ the source term. Γ_D represents the essential boundary (Dirichlet boundary) in which the potential is described as known $g(\mathbf{x})$. Γ_N denotes the natural boundary (Neumann boundary) in which the normal derivative is determined as specified $h(x)$, $\mathbf{x}$ the multidimensional independent variables in the interested domain Ω, Γ_D and Γ_N construct the whole boundary of Ω.

The solution of Eqs. (1)–(3) can be decomposed as

$$u = u_h + u_p \tag{4}$$

where u_h and u_p represent the general and particular solutions of the present problem, respectively. The particular solution u_p satisfies

$$\nabla^2 u_p(\mathbf{x}) = f(\mathbf{x}) \tag{5}$$

but does not necessarily satisfy the boundary conditions. The basic idea of the DRM is to expand the source function $f(x)$ in terms of its values at the interpolation nodes $\mathbf{s}_k$ so that a particular solution of Eq. (5) can be calculated [20]. To evaluate

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