



# Values and tail values at risk of doubly compound inhomogeneous and contagious aggregate loss processes

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## ABSTRACT

This article provides efficient methods based on the saddlepoint approximation for computing the value at risk and the tail value at risk of the doubly compound and perturbed insurer total claim amount. The model is based on a primary counting birth process, for the number of catastrophic events, and on a secondary counting distribution, with possible modification or truncation at zero, for the number of individual losses generated from each catastrophe of the primary process. The considered primary processes are the inhomogeneous Poisson process and the homogeneous contagious Binomial and Negative Binomial processes, which have negative and positive contagions, respectively. The individual claim amounts are independent with a linear combination of Exponential distributions or with a Gamma distribution. The proposed methods are based on the saddlepoint approximation of Lugannani and Rice (1980) [12] and do not require Monte Carlo simulation. They are numerically very accurate, computationally efficient and hence relevant for the actuarial practice.

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## 1. Introduction

The homogeneous compound Poisson process is a standard and widely used stochastic model for the insurer total claim amount or total loss arising from a portfolio of risks over a given period. This model is however subject to various limitations and in particular it appears inappropriate for modelling individual claim amounts arising from naturally occurring catastrophes like hurricanes, floods or earthquakes, which are increasingly affecting the industry. Catastrophic events might also result from the global warming of the atmosphere, which has been observed since the middle of the twentieth century and which is projected to continue. Accurate predictions of aggregate losses arising from catastrophes are therefore essential in the assessment of capitalizations of insurance and reinsurance companies. They are also important in financial markets, where catastrophe bonds are among the most recently traded financial derivatives. In this context, we consider a doubly compound stochastic process for the insurer aggregate loss or claim amount, which takes separately into account the catastrophic events generating the individual claim amounts. This model is based on a primary counting process for the number of catastrophic events and on secondary counting random variables for the number of individual losses generated from each catastrophe. The secondary counting random variables are assumed independent and identically distributed (i.i.d.) and independent of the primary counting process. The primary process and the secondary random variables are assumed independent from the individual financial losses, which are assumed i.i.d. In addition to this, we consider an independent Wiener perturbation process which quantifies other sources of uncertainties related to investment or to claim settlement, for example, when one or more foreign currencies are required for the claim settlement (as typical paths of exchange rates exhibit the characteristics of the Wiener process).

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