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New variable transformations for evaluating nearly singular integrals in 2D boundary element method

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ABSTRACT

This work presents a further development of the distance transformation technique for accurate evaluation of the nearly singular integrals arising in the 2D boundary element method (BEM). The traditional technique separates the nearly hypersingular integral into two parts: a near strong singular part and a nearly hypersingular part. The near strong singular part with the one-ordered distance transformation is evaluated by the standard Gaussian quadrature and the nearly hypersingular part still needs to be transformed into an analytical form. In this paper, the distance transformation is performed by four steps in case the source point coincides with the projection point or five steps otherwise. For each step, new transformation is proposed based on the approximate distance function, so that all steps can finally be unified into a uniform formation. With the new formulation, the nearly hypersingular integral can be dealt with directly and the near singularity separation and the cumbersome analytical deductions related to a specific fundamental solution are avoided. Numerical examples and comparisons with the existing methods on straight line elements and curved elements demonstrate that our method is accurate and effective.

1. Introduction

Near singularities are involved in many boundary element method (BEM) analyses of engineering problems, such as the thin and shell-like problems [1–3,6], the crack problems [7], the contact problems [8], as well as the sensitivity problems [9]. Accurate and efficient evaluation of nearly singular integrals is crucial for successful implementation of BEM analyses. A near singularity arises in BEM when a source point is close to but not on the integration elements. Although those integrals are really regular in nature, they cannot be evaluated accurately by the standard Gaussian quadrature. This is the so-called boundary layer effect in BEM. The boundary layer effect comes from the properties of fundamental solutions and their derivatives. The denominator, the distance between the source and the field point, is close to zero but not zero. The difficulty encountered in the numerical evaluation mainly results from the fact that the integrands of nearly singular integrals vary drastically with the distance.

Effective computation of nearly singular integrals has received intensive attention in recent years. Various numerical techniques have been developed to remove the near singularities, such as rigid body displacement solutions [10], global regularization [4,5,11–14], semi-analytical or analytical integral formulas [15,16], the sinh transformation [17–19], polynomial transformation [20], adaptive subdivision method [21–23], distance transformation technique [24–27], the $L_1^{-(1/5)}$ transformation [28] and the PART method [29]. Most of them benefit from the strategies for computing singular integrals. Among those techniques, the distance transformation technique seems to be a more promising method for dealing with different orders of nearly singular integrals. However, the traditional technique separates the nearly hypersingular integral into two parts with the aid of an introduced term having the similar hypersingular properties: a near strong singular part and a near hypersingular part. The near strong singular part with the one-ordered distance transformation can be evaluated by the standard Gaussian quadrature and the near hypersingular part still needs to be transformed into an analytical form. This is because the two distance transformations in Refs. [24–27] for the nearly hypersingular integral are not effective.

To cope with the above problems, a number of new transformations are introduced based on the approximate distance function to deal with the nearly hypersingular integral directly. Hence, the near singularity separation and the cumbersome formula deductions of the near hypersingular part in Refs. [24–27] are no longer required. We first take four steps to analyze the transformation when the distance between the source point and the projection point equals zero, and five steps otherwise. In each

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