Engineering Analysis with Boundary Elements 35 (2011) 527–532

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Deriving exact Green's functions and integral formulas for a thermoelastic wedge

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A R T I C L E I N F O

Article history:
Received 12 February 2010
Accepted 18 August 2010
Available online 15 September 2010

Keywords:
Theorem
Green's functions
Thermoelasticity
Heat conduction
Elasticity
Thermoelastic influence functions
Elastic volume dilatation
Wedge
Quarter-space
Half-space

A B S T R A C T

In this paper an exact Green's function and a Poisson-type integral formula for a boundary-value problem (BVP) for a thermoelastic wedge are derived in elementary functions. The thermoelastic displacements are generated by a heat source distributed in the inner points of the wedge. On the boundary half-planes the temperature is given, the mechanical boundary conditions being locally-mixed and homogeneous. Similar results for a quarter-space and for a half-space as particular cases of the wedge also are included. According to the well-known analogy between thermoelasticity and poroelasticity, the obtained results can be extended to poroelasticity problems, by making use of changes in respective constants. This paper introduces an approach for deriving new thermoelastic and poroelastic influence functions and Poisson-type integral formulas in closed form not only for wedge, quarter-space and half-space but also for many other canonical domains.

1. Introduction

Green's function plays a key role in finding solutions in integrals for boundary-value problems (BVPs) in the different fields of mathematical physics. This includes thermoelasticity, a combination of the theory of heat conduction and theory of elasticity. In this paper, we prove a theorem on deriving new Green's functions and Poisson-type integral formula for the thermoelastic wedge $V(0\le r<\infty;\ 0\le\varphi\le\alpha; -\infty<z<\infty)$; $\alpha=\pi/n; n=1,2,3,\ldots,$ where cylindrical coordinates (r,φ,z) are used. Thermoelastic displacements are generated by an inner heat source with the homogeneous locally-mixed mechanical boundary conditions and with the non-homogeneous Dirichlet's boundary heat condition (temperature is prescribed on the boundary). The results obtained can be used for determining the solution to the BVPs in thermoelasticity using different methods, including boundary element method (BEM) [1]. To prove this theorem, we used the basic equations in thermoelastostatics as well as the general Green's integral formula.

1.1. Basic equations in stationary thermoelasticity

To date, a number of theories on thermoelasticity have been developed and described in the classical scientific literature [2–5]. However, many new developments of thermoelasticity and many references are included in [6]. The most widely used theory in practical calculations is the theory of stationary thermal stresses, i.e., the theory of uncoupled thermoelasticity, where the temperature field does not depend on the field of elastic displacements, and, where inertial terms can be ignored. According to this theory, the formulation of the BVP consists of non-homogeneous Lame's equations

$$\mu\left(\nabla^2 u_\rho-\rho^{-2}u_\rho-2\rho^{-2}\partial u_\psi/\partial\psi\right)+(\lambda+\mu)\partial\theta/\partial\rho-\gamma\partial T/\partial\rho=0;$$
$$\mu\left(\nabla^2 u_\psi-\rho^{-2}u_\psi+2\rho^{-2}\partial u_\rho/\partial\psi\right)+(\lambda+\mu)\rho^{-1}\partial\theta/\partial\psi-\gamma\rho^{-1}\partial T/\partial\psi=0;$$
$$\mu\nabla^2 u_\xi+(\lambda+\mu)\partial\theta/\partial\xi-\gamma\partial T/\partial\xi=0 \qquad (1)$$

with the respective mechanical boundary conditions. In Eq. (1) the following notation are used: (ρ,ψ,ξ) are the cylindrical coordinates of the point in which the displacements u_ρ,u_ψ,u_ξ are determined; $\theta=\partial u_\rho/\partial\rho+\rho^{-1}u_\rho+\rho^{-1}(\partial u_\psi/\partial\psi)+\partial u_\xi/\partial\xi$ is the thermoelastic volume dilatation, created by temperature T; $\nabla^2=\partial^2/\partial\rho^2+\rho^{-1}\partial/\partial\rho+\rho^{-2}\partial^2/\partial\psi^2+\partial^2/\partial\xi^2$ is the Laplace's differential operator; λ, μ are the Lame's constants of elasticity; $\gamma=\alpha_t(2\mu+3\lambda)$ is the thermoelastic constant; α_t is the coefficient of the linear thermal expansion. The temperature field in Eq. (1) can be determined from the BVP in heat condition that consists of Poisson's equation

$$\nabla^2 T(N)=-a^{-1}F(N);\quad N\in V \qquad (2)$$

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doi:10.1016/j.enganabound.2010.08.016