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Applications of differential subordination to certain subclasses of p-valent meromorphic functions involving a certain operator

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ABSTRACT

By making use of the method of differential subordination, we investigate some properties of certain classes of p-valent meromorphic functions, which are defined by means of a certain operator.

1. Introduction

For any integer $m > -p$, let $\sum_{p,m}$ denote the class of all meromorphic functions f normalized by:

$$f(z) = z^{-p} + \sum_{k=m}^{\infty} a_k z^k \quad (p \in \mathbb{N} = \{1, 2, \ldots\}), \tag{1.1}$$

which are analytic and p-valent in a punctured unit disk $U^* = \{z : z \in \mathbb{C} \text{ and } 0 < |z| < 1\} = U \setminus \{0\}$. For convenience, we write $\sum_{p,-p+1} = \sum_p$. If f and g are analytic functions in U, we say that f is subordinate to g, written $f \prec g$ if there exists a Schwarz function w, which (by definition) is analytic in U with $w(0) = 0$ and $|w(z)| < 1$ for all $z \in U$, such that $f(z) = g(w(z)),\ z \in U$. Furthermore, if the function g is univalent in U, then we have the following equivalence (see [1–3]):

$$f(z) \prec g(z) \Leftrightarrow f(0) = g(0) \quad \text{and} \quad f(U) \subset g(U).$$

For functions $f(z) \in \sum_{p,m}$ given by (1.1) and $g(z) \in \sum_{p,m}$ given by $g(z) = z^{-p} + \sum_{k=m}^{\infty} b_k z^k$, the Hadamard product of $f(z)$ and $g(z)$ is given by:

$$(f * g)(z) = z^{-p} + \sum_{k=m}^{\infty} a_k b_k z^k = (g * f)(z). \tag{1.2}$$

For complex numbers $\alpha_1, \alpha_2, \ldots, \alpha_l$ and $\beta_1, \beta_2, \ldots, \beta_s (\beta_j \notin Z_0^- = \{0, -1, -2, \ldots\}; j = 1, 2, \ldots, s)$, we define the generalized hypergeometric function ${}_lF_s(\alpha_1, \ldots, \alpha_l; \beta_1, \ldots, \beta_s; z)$ (see, for example, [4]) by the following infinite series:

$$ {}_lF_s(\alpha_1, \ldots, \alpha_l; \beta_1, \ldots, \beta_s; z) = \sum_{k=0}^{\infty} \frac{(\alpha_1)_k \cdots (\alpha_l)_k}{(\beta_1)_k \cdots (\beta_s)_k (1)_k} z^k \quad (l \le s+1; s, l \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}; z \in U), \tag{1.3}$$

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