Mathematical and Computer Modelling 54 (2011) 1440–1452

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Fractional functional differential equations with causal operators in Banach spaces[☆]

Ravi P. Agarwal[a], Yong Zhou[b,*], JinRong Wang[c], Xiannan Luo[b]

[a] Department of Mathematics, Florida Institute of Technology, FL 32901, USA

[b] Department of Mathematics, Xiangtan University, Xiangtan, Hunan 411105, PR China

[c] Department of Mathematics, Guizhou University, Guiyang, Guizhou 550025, PR China

ARTICLE INFO

Article history:
Received 18 March 2011
Accepted 12 April 2011

Keywords:
Fractional functional differential equations
Causal operators
Cauchy problem
Measure of noncompactness

ABSTRACT

In this paper, we study the fractional functional differential equations with causal operators in an arbitrary separable Banach space. By means of the techniques of the measure of noncompactness, the existence and continuation of solutions are given. Further, some topological properties of solution sets are discussed and the existence of optimal solutions of the associated control problem are presented. An example is given to illustrate the result.

1. Introduction

The study of functional equations with causal operators has seen a rapid development in the past few years and some results such as the existence, stability and control are assembled in a monograph of Corduneanu [1]. The term causal operators is adopted from the engineering literature and the theory of these operators has the powerful quality of unifying ordinary differential equations, integrodifferential equations, differential equations with finite or infinite delay, Volterra integral equations, and neutral functional equations, to name but a few. A pioneering work has been reported by Corduneanu [2], Drici et al. [3–5], and Lupulescu [6]. For more details, the reader can refer to the recent monograph of Lakshmikantham et al. [7] and the papers of Devi [8], and Obukhovskii and Zecca [9].

On the other hand, fractional differential equations have proved to be valuable tools in the modeling of many phenomena in various fields of engineering, physics and economics. It draws a great application in nonlinear oscillations of earthquakes, many physical phenomena such as seepage flow in porous media and in fluid dynamic traffic model. Actually, fractional differential equations are considered as an alternative model to integer differential equations. For more details, one can see the monographs of Diethelm [10], Kilbas et al. [11], Lakshmikantham et al. [12], Miller and Ross [13], Podlubny [14], and Tarasov [15]. Recently, fractional differential equations (inclusions) and optimal controls in Banach spaces have been studied by Agarwal et al. [16,17], Nieto et al. [18–21], Balachandran et al. [22,23], Benchohra et al. [24], El-Borai [25], Henderson and Ouahab [26], Hernández [27], Mophou and N'Guérékata [28], Wang et al. [29,30], and Zhou et al. [31–36].

To our knowledge, fractional functional differential equations with causal operators in Banach spaces have not been studied extensively. There are only relatively scarce results [37] on fractional differential equations with causal operators.

☆ Research supported by the Natural Science Foundation of China (10971173), Tianyuan Special Funds of the National Natural Science Foundation of China (11026102) and Key Projects of Science and Technology Research in the Ministry of Education (211169).

* Corresponding author.
E-mail addresses: agarwal@fit.edu (R.P. Agarwal), yzhou@xtu.edu.cn (Y. Zhou), wjr9668@126.com (J. Wang), xnluo@xtu.edu.cn (X. Luo).

0895-7177/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2011.04.016