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# Multi-scale based stochastic damage evolution

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#### ARTICLE INFO

Article history: Available online 15 December 2010

Keywords: Multi-scale Damage model Stochastic evolution Random field

### ABSTRACT

In the present work, the stochastic damage evolution is developed by the random extension of the homogenization based multi-scale damage model. The energy bridging theorem is introduced as the bridging vehicle between micro and macro scales. And then the probability density evolution method (PDEM) and the generalized density evolution equation (GDEE) are introduced to describe the nonlinear transportation of randomness between scales. It is observed from the numerical results that not only the randomness induced stress field oscillation but also the stochastic damage evolution are well reproduced by the proposed framework.

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Failure Analysis

#### 1. Introduction

Damage mechanics is equipped with a multi-scale nature. The dynamics of micro-cracks, which is the cause of the damage, is often defined in micro-scale. And the resultant continuum damage evolution is usually considered in the macroscopic analysis of structures. On the other hand, the classic continuum damage mechanics is phenomenological. That is to say, the damage evolution function, which is the core part of the theory, are not related to specific microstructural parameters and local defects. Thus the damage evolution based on multi-scale framework has been investigated in recent years. Lee et al. [1] developed the voronoi cell finite element model (VCFEM) in the damaged region. A nonlocal damage theory based on asymptotic homogenization was proposed by Fish et al. [2] to account for damage effects in heterogeneous media. However, the microscopic failure are also considered by damage evolution rather than explicit modeling of defects according to this method. Li and Ren [3] proposed another class of multi-scale damage methods according to asymptotic expansion based homogenization. The continuum damage evolution is directly informed by the propagation of defects in micro-scale via energy bridging theorem. This approach also avoids the tedious simulation of characteristic functions in the conventional asymptotic type method.

A close observation of micro-structures reveals its inherent randomness which has been draw intensive attention due to its influence on the micro as well as the macro performances. The pioneering researchers adopted the stochastic perturbation method for the stochastic extension of the asymptotic based homogenization to consider the transformation of randomness in micro and macro scales. The perturbation based stochastic homogenization provides acceptable approximations of the low order stochastic characteristics such as the expectation and variance if the random variation of the system properties is relatively small. Different methods are proposed based on the orders of the asymptotic expansion. Such as the first-order method [4], the second order method [5] and the higher order method [6]. It is clearly that with the above approaches the stochastic homogenization have not been well developed yet for the nonlinear structural problems. Thus an alternative attempt for the nonlinear stochastic problems is proposed based on a point of view of probability density functions

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