



The Cauchy problem for the modified Kawahara equation in Sobolev spaces with low regularity

Wei Yan^{a,*}, Yongsheng Li^a, Xingyu Yang^b

^a Department of Mathematics, South China University of Technology, Guangzhou, Guangdong 510640, PR China

^b School of Business Administration, South China University of Technology, Guangzhou, Guangdong 510640, PR China

ARTICLE INFO

Article history:

Received 26 September 2009

Received in revised form 27 March 2011

Accepted 28 March 2011

Keywords:

Cauchy problem

Global well-posedness

Modified Kawahara equation

Sobolev spaces

Bourgain spaces

ABSTRACT

This paper is concerned with the Cauchy problem of the modified Kawahara equation. By using the Fourier restriction norm method introduced by Bourgain, and using the I -method as well as the L^2 conservation law, we prove that the modified Kawahara equation is globally well-posed for the initial data in the Sobolev space $H^s(\mathbf{R})$ with $s > -\frac{3}{22}$.

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1. Introduction

This paper is devoted to study the initial-value problem of the modified Kawahara equation

$$\partial_t u + \alpha \partial_x^5 u + \beta \partial_x^3 u + \gamma \partial_x u + \mu \partial_x(u^3) = 0, \quad x \in \mathbf{R}, t > 0, \quad (1.1)$$

$$u(x, 0) = u_0(x), \quad (1.2)$$

where $\alpha \neq 0$, β and γ are real numbers and μ is a complex number.

Model (1.1) is called the modified Kawahara equation which arises in the study of water waves with surface tension. When $\alpha = 0$, $\beta \neq 0$ and $\mu \neq 0$ in (1.1), we have the modified KdV equation which has been extensively studied by many authors, for instance, see [1–7]. In [3], the authors proved that the modified KdV equation is locally well-posed for the initial data in $H^s(\mathbf{R})$ with $s \geq \frac{1}{4}$. In [4], the authors proved that

$$\|\partial_x(u^3)\|_{X_{s,b-1}} \leq C \|u\|_{X_{s,b}}^3$$

for $s < \frac{1}{4}$ and $b \in \mathbf{R}$, is invalid, where $X_{s,b}$ is the standard Bourgain space associated to the modified KdV equation. In [5], the authors proved that the regularity requirement, $s \geq \frac{1}{4}$, is sharp. In [1], the authors established the global well-posedness of the modified KdV equation in the range $s > \frac{1}{4}$. In [6], the authors considered the local and global Cauchy problems for the generalized Korteweg–de Vries equation

$$\partial_t u + \partial_x^3 u + u^k \partial_x u = 0, \quad k \geq 4, k \in \mathbf{N}^+,$$

* Corresponding author.

E-mail addresses: yanwei.scut@yahoo.com.cn (W. Yan), yshli@scut.edu.cn (Y. Li), xyyangmath@gmail.com (X. Yang).