International Journal of Thermal Sciences 50 (2011) 2104–2116

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Heat diffusion vs. wave propagation in solids subjected to exponentially-decaying heat source: Analytical solution

Tung T. Lam*, Ed Fong [1]

The Aerospace Corporation, El Segundo, CA 90245, USA

ARTICLE INFO

Article history:
Received 22 November 2010
Received in revised form
2 May 2011
Accepted 4 May 2011
Available online 22 June 2011

Keywords:
Laser heating
Superposition method
Solution structure theorem
Hyperbolic heat conduction
Parabolic heat conduction

ABSTRACT

The classical parabolic heat conduction equation based on Fourier's law of heat conduction and the hyperbolic heat conduction equation based on the Cattaneo–Vernotte (C–V) constitutive relation are compared in this study. The present investigation considers the effect of thermal diffusion and wave propagation in solids subjected to a time-varying and spatially-decaying laser irradiation. The incoming energy is simulated as internal heat generation inside the medium. Temperature profiles are presented as closed-form series solutions for both models by using the method of superposition and the solution structure theorem. Results demonstrate that temperatures, based on the classical diffusion theory, are under-predicted at small and moderate times as compared to the wave model under high heat flux and rapid change in temperature situations. The C–V model predicts a higher peak temperature inside the medium and forms a small wave beneath the incident surface due to the lagging behavior in energy transport when compared to the diffusion model at extremely short time periods. The diffusion model may show an overall smaller temperature in the beginning; however, it over-predicts the peak temperature as compared to the results with the wave model at large times.

1. Introduction

Heat transfer in solids based on the macroscopic diffusion theory is widely accepted in the scientific and engineering community. The empirical law of diffusive heat conduction originated from Biot but is generally credited to the French mathematical physicist, Joseph Fourier, who published the theory in his book titled, *Théorie Analytique de la Chaleur*, in 1822 [1]. The Fourier law governing thermal energy transport in solids states that the rate of heat flow in a given direction is proportional to the area normal to the direction of travel and to the temperature gradient in the same direction. Mathematically, for a one-dimensional analysis with heat flow in the x-direction, the heat flux can be written as

$$q^* = -k\frac{\partial T^*}{\partial x^*} \tag{1}$$

The classical Fourier parabolic heat equation has been used for all analyses until the 1950s even though it assumes that thermal energy travels within the solid medium at a non-physical infinite speed. This is a valid assumption for typical applications, but it breaks down in situations that include low temperature conditions or engineering applications with high-power for short duration. In 1958, Cattaneo [2] and Vernotte [3] proposed a new form of the heat conduction equation in separate investigations by adding a relaxation term to account for the increase in the heat flux vector due to phonon collisions in a duration of the mean free time, denoted by τ. The equation takes the form as

$$\tau\frac{\partial q^*}{\partial t^*} + q^* = -k\frac{\partial T^*}{\partial x^*} \tag{2}$$

Eq. (2) is the classical thermal wave model in which the mean free time is referred to as the relaxation time and is defined as the ratio of the effective mean free path to the phonon speed (the speed of sound). As a result of this modification, the original energy conservation equation, assuming parabolic heat conduction, is transformed into a hyperbolic wave equation. One can envision that if $\tau = 0$, which denotes either a zero mean free path or an infinite speed in phonon collisions, the thermal wave equation, Eq. (2), is reduced to the classical Fourier law, Eq. (1).

The hyperbolic heat conduction equation has been applied to the study of many practical engineering applications. Chan et al. [4] studied the effect of finite propagation heat velocity on the temperature rise of crystallites in exothermic catalytic reactions.

* Corresponding author. Tel.: +1 310 336 5408; fax: +1 310 336 2270.
E-mail addresses: tung.t.lam@aero.org (T.T. Lam), ed.fong@aero.org (E. Fong).
[1] Tel.: +1 310 336 3437; fax: +1 310 336 2270.

1290-0729/$ – see front matter © 2011 Elsevier Masson SAS. All rights reserved.
doi:10.1016/j.ijthermalsci.2011.05.006