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# A high resolution nonstandard FDTD method for the TM mode of Maxwell's equations $\protect{\scalar}$

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#### 1. Introduction

Recently, several algorithms have been developed to solve Maxwell's equations with the interest of providing accurate predictions in complex electromagnetic phenomena. In particular, the understanding of the physical and chemical properties of nanostructures is now an important area of study, see [1]. Light scattering simulations are a support tool to determine the optimum nanoparticle size and their distribution in the media. Some of the well-known computational techniques for these simulations are the finite element method (FEM) and the finite difference time domain method (FDTD), see [2,3]. The FDTD method, originally proposed by Kane Yee in 1966 [4], is a second-order explicit scheme that does not require matrix inversion in contrast to the competing FEM. The FDTD method is often the method of choice for the previous reasons and for its accurate numerical results in problems with complex shapes and structural properties, see [5]. However, FDTD computation of light scattering by small particles implies a high computational cost and even though the FDTD algorithm is massively parallelizable is important to reduce significantly the computational cost. In this direction, a paper by Cole [6] presented a new FDTD scheme to solve Maxwell's equations with fewer grid points which maintains the accuracy of the conventional FDTD method. Cole's scheme is based on Mickens' ideas for nonstandard finite difference methods [7], where discrete representations for derivatives have non-trivial denominator functions. For example, a nonstandard central difference approximation for the first derivative of a differentiable function *f* is  $f'(x) = (f(x + h/2) - f(x - h/2))(\phi(h))^{-1}$ , *h* being the distance between two grid nodes and  $\phi$ , a step-denominator function, satisfying  $\phi(h) = h + O(h)$ .

The nonstandard FDTD method developed by Cole has been applied with high computational efficiency to several problems, some examples can be found in [8–10]. However, the nonstandard FDTD method presents instabilities in some cases due to a correction function which depends on the phase term. In order to overcome this issue, we propose an alternative correction function which does not consider the phase constant.

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#### ABSTRACT

In this work, a high resolution nonstandard finite-difference time-domain method for the transverse magnetic (TM) mode of 2D-Maxwell's equations is derived. Stability and convergence analysis and numerical studies are presented for the proposed method. The accuracy of this numerical scheme is validated by simulating a light scattering cross-section by a perfectly-conducting circular cylinder.

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