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Onset of convective rolls in a circular porous duct with external heat transfer to a thermally stratified environment

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ABSTRACT

A horizontal circular duct filled with a fluid saturated porous medium is studied. The external wall is assumed to exchange heat with an external environment thermally stratified in the vertical direction. The external heat transfer is modeled through a third kind boundary condition, and a Biot number associated with the external heat transfer coefficient is defined. The linear stability of the basic state where the velocity field is zero is studied numerically. The condition of neutral stability is determined, by solving the system of elliptic governing equations for the disturbances through a Galerkin finite-element method. The neutral stability curves, together with the critical values of the wave number and of the Rayleigh number, are obtained for different values of the Biot number. The case of a duct with a finite axial length, having impermeable and thermally insulated axial boundaries, is considered. On increasing the length-to-radius aspect ratio, the transition from a two-dimensional to a three-dimensional pattern of instability at the onset of convection is described.

1. Introduction

The thermal instability in a fluid saturated porous medium has been the subject of a wide literature in the last six decades. The pioneering studies in this field were conducted in refs. [1,2]. These papers are relative to a plane horizontal porous layer with isothermal boundaries. The stability was analyzed for a basic state with vanishing velocity and a vertical temperature gradient oriented downward. This classical problem, well known either as the Darcy–Bénard problem or the Horton–Rogers–Lapwood problem has been thoroughly studied and extended to different models of momentum transfer in the porous medium, beyond Darcy's law, as well as to different types of velocity and temperature boundary conditions. Reviews of the most important results for the instability in a porous medium with heating from below are available in refs. [3–5]. For the general theory of convection in porous media we refer the reader to refs. [6–9].

Among the many branches of the theoretical research originated by the classical Darcy–Bénard problem, we focus here on the investigations about the onset of convection in porous cylinders, and on the role played by the boundary conditions expressing a finite thermal conductance. These thermal boundary conditions, usually called Robin boundary conditions or boundary conditions of the third kind, imply the emergence of an additional governing parameter: the Biot number. The studies of the thermoconvective instability in porous cylinders have mainly regarded vertical cylinders. Zebib [10] analyzed the onset conditions of the thermal instability in a vertical porous cylinder with isothermal and impermeable horizontal boundaries, with an adiabatic and impermeable lateral boundary. More recently, Haugen and Tyvand [11] revisited Zebib's problem on assuming a temperature distribution, at the lateral vertical wall, changing linearly in the vertical direction. This boundary condition models the heat transfer to an external environment thermally stratified in the vertical direction. In particular, the model adopted in ref. [11] presumes an infinitely large heat transfer coefficient to the external environment, resulting in a Dirichlet boundary condition for the temperature disturbance. This model of the thermal boundary conditions for the vertical wall was generalized by Nygård and Tyvand [12], with reference to a two-dimensional rectangular cavity, and by Nygård and Tyvand [13] for a vertical porous cylinder. The generalization consists in transforming the Dirichlet boundary condition into a Robin boundary condition, thus assuming a finite heat transfer coefficient to the external environment. We mention that the use of Robin boundary conditions for the temperature field were explored, in the framework of the thermoconvective instability, by Kubitschek and Weidman [14] in order to model the bottom heating in a vertical

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