International Journal of Thermal Sciences 50 (2011) 1428–1444

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Effective thermal conductivity of periodic composites with highly conducting imperfect interfaces

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A R T I C L E I N F O

Article history:
Received 23 October 2010
Received in revised form
22 February 2011
Accepted 8 March 2011
Available online 9 April 2011

Keywords:
Thermal properties
Particles-reinforced composites
Interface
Transport properties
Size effects

A B S T R A C T

The purpose of this work is to determine the effective conductivity of periodic composites accounting for highly conducting imperfect interfaces between the matrix and inclusions phases and to study the dependencies of the effective conductivity on the size and distribution of inhomogeneities in the matrix phase in different cases: squared, hexagonal, cubic and random inclusion distributions. The local solution of the periodic conduction problem is found in Fourier space by using the Green operators and closed-form expressions of factors depending on the size and shape of the inclusions. The numerical results of size-dependent effective thermal conductivity are finally compared with an analytical estimation obtained from the generalized self-consistent model. The method elaborated and results provided by the present work are directly applicable to other physically analogous transport phenomena, such as electric conduction, dielectrics, magnetism, diffusion and flow in porous media and to the mathematically identical phenomenon of anti-plane elasticity.

1. Introduction

Recently, different works have been devoted to study the size-dependent mechanical behavior in nanosystems incorporating surface/interface energies. Indeed, when the inclusion size is diminished to the nano-scale, due to the large surface-to-volume ratio, the matrix–inclusion interface energy can no longer be neglected. This fact has been emphasized and exploited in recent investigations on nanomaterials and nano-structural elements (see, e.g. [11,24]). In this context, in order to estimate the size-dependent overall elastic properties of nanocomposites and nano-structural elements accounting for the surface/interface energies, the classical perfect interface is modified by adopting a coherent interface model in which the displacement vector field is continuous across an interface while the stress vector field is discontinuous across the same interface ([2,29,31]). The thermal conduction counterpart of the interface stress and energy model is the highly conducting (HC) interface model, which is the subject of the present paper. More precisely, according to this imperfect interface model, the temperature is continuous across this interface but the normal heat flux component is discontinuous across the same interface due to the possibility of having a surface heat flux along the interface whose surface energy conservation equation gives rise to the generalized Young–Laplace equation. These interface conditions for the HC interface model are completely contrary to the ones of the well-known Kapitza interface thermal resistance model which has been recognized to be of a great theoretical and practical importance in physics and materials science (see e.g. [17]). By accounting for the thermal resistance appearing at the interface between two bulk media, the Kapitza interface thermal resistance model stipulates that the temperature suffers a jump across the interface while the normal heat flux component is continuous across the same interface and usually taken to be proportional to the temperature jump. Thus, the HC interface model can be viewed as dual with respect to the Kapitza interface thermal resistance model. The physical background of both HC interface model and thermal resistance interface model can be clarified by considering the general imperfect interface model in which a very thin interphase of uniform thickness is situated between two bulk phases. By applying an asymptotic approach to this interphase to obtain appropriate temperature and normal heat flux component jump conditions for an interface of zero thickness replacing the interphase, Sanchez-Palencia [30] and Pham Huy and Sanchez-Palencia [28] showed that the general imperfect interface model reduces to the HC interface model or the thermal resistance interface model according as the interphase is highly conducting or slowly conducting with respect to the surrounding phases. In other words, the HC interface model and the thermal resistance interface model may be considered as the two limiting cases of the general imperfect interface model.

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doi:10.1016/j.ijthermalsci.2011.03.009