



## Theoretical conjugate heat transfer analysis in a parallel flat plate microchannel under electro-osmotic and pressure forces with a Phan-Thien-Tanner fluid

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### ARTICLE INFO

#### Article history:

Received 26 August 2010

Received in revised form

26 January 2011

Accepted 28 January 2011

Available online 1 March 2011

#### Keywords:

Electro-osmotic

Phan-Thien-Tanner

Conjugate heat transfer

Joule heating

Microchannel

### ABSTRACT

In this paper we solve, numerically and asymptotically, the steady-state analysis of a conjugate heat transfer process in an electro-osmotic and fully developed laminar flow including Joule heating effects. In addition, the viscoelastic fluid obeys the simplified Phan-Thien-Tanner (SPTT) constitutive equation. Taking into account the finite thermal conductivity of the micro-channel wall, the dimensionless temperature profiles in the fluid and solid wall have been obtained as functions of the dimensionless parameters involved in the analysis: a conjugate parameter,  $\alpha$ , which represents the competition between the longitudinal conductive heat in the micro-channel wall to the convective heat transfer in the fluid;  $\epsilon De_k^2$ , a parameter that describes the viscoelastic behavior of the fluid; the well-known Peclet number,  $Pe$ ; a normalized power generation term,  $\mathcal{A}$ , being the ratio of heat flux from the external wall to the Joule heating (and smaller or equal to unity); the ratio of pressure to the electro-osmotic forces,  $\Gamma$ ; and the aspect ratios of the micro-channel and the solid wall,  $\beta$  and  $\epsilon$ , respectively. The results for the temperature fields, in the fluid and micro-channel wall show a strong dependence of the above dimensionless parameters, therefore, this set of parameters controls directly the thermal performance of this micro-channel model.

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### 1. Introduction

Microfluidic and lab-on-a-chip devices are used in the handling of biomedical and chemical analysis. These integrated, miniaturized devices offer many advantages over conventional laboratory bench-top analytical instruments, such as increased efficiency, throughput, portability, and reduced analysis time, reagent consumption, and cost [1–3]. Generally, an electric field is often applied to induce electro-osmotic flow of liquids and electrophoretic motion of particles in microfluidic devices. Originally, the electrokinetic transport operates as a combination of two mechanism drivers: electrophoresis and electro-osmosis; the first refers to the migration of charged solutes (e.g. ions, macromolecules of DNA) in electrolyte, and the second one gives the movement of a volume of an aqueous solution adjacent to a charged solid surface when an external electric field is applied [4]. Advances in microfluidic devices make possible a complete analysis of fluids in the biochemistry area in a single fabricated chip; therefore it is fundamental to understand the characteristics of fluid flow in microchannels to have an optimum design and precise control of microfluidics devices [5].

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The physics of electrokinetic phenomena, specifically in the study of electro-osmotic and electrophoretic transport has been reviewed extensively in the specialized literature. The fundamental hydrodynamic of electro-osmotic and electrophoretic flow is detailed by Li [2], Masliyah and Bhattacharjee [6] and Karniadakis et al. [7]. While electrokinetic methods can greatly simplify the flow control and species transport in microfluidic systems, the Joule heating is inevitable in the liquid when an axial electrical field is applied to generate the electrokinetic flow [8–14]. This internal heat source can lead to significant increases in the liquid temperature. In this context, the Joule heating effect on the electro-osmotic flows has been widely studied by Xuan et al. [13,14], who analyzed the conjugate heat transfer problem and thermal end effects. Their model accounts for the dynamic coupling effects of Joule heating on the temperature, the electrical double layer, the applied electrical potential and the flow fields in a capillary tube from reservoir to reservoir. These fields are strongly coupled because the corresponding properties are temperature dependent. Tang et al. [4,10,11] carried out the analysis of the Joule heating effect on the electro-osmotic flow; the mathematical model for describing the Joule heating in an electro-osmotic flow including the Poisson–Boltzmann equation, the modified Navier–Stokes equations and the energy equation was numerically treated. The system of equations of the above model is coupled through the temperature-dependent liquid dielectric constant,