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Thermally developing microtube gas flow with axial conduction and viscous dissipation

A. Aziz*, Nick Niedbalski

Department of Mechanical Engineering, School of Engineering and Applied Science, Gonzaga University, Spokane, WA 99258, USA

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This work is dedicated to the memories of first author's younger brother, Mohammed Iqbal, CSP and his wife Iffat Iqbal whose lives were brutally terminated by the rebel forces in East Pakistan in 1971.

Keywords:
Thermally developing gas flow
Microtube
Axial conduction
Viscous dissipation
Second-order slip
Temperature jump

ABSTRACT

A comparative study of the first and second order slip flow models on the thermal development of dilute gas flow in a microtube with axial conduction and viscous dissipation has been performed. The velocity profiles for the two models have been derived analytically assuming the flow to be fully developed hydrodynamically. The energy equation is solved numerically for a constant wall temperature with velocity slip and temperature jump conditions at the wall. Analytical solutions for the asymptotic values of the mean temperature and Nusselt number are given for the second-order slip flow model in terms of Brinkman number *Br*, Knudsen number *Kn*, temperature jump parameter κ, and slip parameter ρ_s. The effect of these parameters and the Peclet number *Pe* on the development of local mean fluid temperature and the local Nusselt number are displayed graphically and compared with the results available in the literature. For the fully developed hydrodynamic condition, the second-order slip model predicts higher velocities in the central region of the flow but the trend is reversed near the wall region. For fixed values of *Pe*, *Kn*, κ, and three different *Br*, the second order model predicts higher fluid temperatures compared with those predicted by the first-order model. For thermally developing flow, the second order model in general predicts significantly higher mean fluid temperatures (maximum of 21% in one case), compared with those predicted by the first-order model. However, the difference in the local Nusselt number between the two slip models is small.

1. Introduction

Laminar heat transfer in macro tubes is a mature topic which is discussed in all heat transfer textbooks and thousands of research papers. However, this vast body of knowledge, which is based in the continuum flow model, is inadequate for the correct characterization of heat and fluid flow characteristics in tubes of extremely small diameters (microns). The microtube geometry appears in many microscale devices that are currently in use in engineering and medical applications such as micro heat sinks, spacecraft thermal control, micro biochips, micro reactors for biological cells, and control and monitoring of minute amounts of medicines. It is therefore not surprising that a significant amount of research effort has been devoted in recent years to understand fluid flow and heat transfer in microtubes. The physics of the flow in microdevices and the progress made on this topic is described by Gad-el-Hak [1].

The parameter which is used to draw the distinction between the continuum and free molecular gas flows is the Knudsen number *Kn*, defined as the ratio of mean free path of molecules λ to the characteristic length of the flow *L*. The flow is classified as continuum regime ($Kn < 0.001$), slip flow regime ($0.001 < Kn < 0.1$), transition regime ($0.1 < Kn < 10$), and free molecule flow ($Kn > 10$). Gas flows in microtubes typically occur in the slip flow regime. For this range, a hybrid flow model has been proposed in which the conservation equations for the continuum model are retained but the traditional conditions of no slip and no temperature jump are replaced by a slip velocity and a finite temperature jump at the gas–solid interface. This hybrid model forms the basis of many theoretical studies in recent years. Because experimental work on gas flows in microtubes requires sophisticated equipment and is challenging [2], analytical and numerical approaches continue to remain the prediction tools of choice for such flows.

The slip velocity is modeled by considering the molecular momentum transport at the fluid–solid interface. The first-order slip velocity model proposed by Maxwell [3] shows that the slip velocity is proportional to the shear stress at the surface. The Maxwell model has been verified experimentally by Tretheway and

* Corresponding author. Tel.: +1 509 313 3540.
E-mail address: aziz@gonzaga.edu (A. Aziz).

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