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Natural convection of nanofluids in a shallow cavity heated from below

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ABSTRACT

This paper reports an analytical and numerical study of natural convection in a shallow rectangular cavity filled with nanofluids. Neumann boundary conditions for temperature are applied to the horizontal walls of the enclosure, while the two vertical ones are assumed insulated. The governing parameters for the problem are the thermal Rayleigh number, *Ra*, the Prandtl number Pr, the aspect ratio of the cavity, *A* and the solid volume fraction of nanoparticles, Φ. For convection in an infinite layer ($A \gg 1$), analytical solutions for the stream function and temperature are obtained using a parallel flow approximation in the core region of the cavity and an integral form of the energy equation. The critical Rayleigh number for the onset of supercritical convection of nanofluids is predicted explicitly by the present model. Furthermore, a linear stability analysis of the parallel flow solution is studied and the threshold for Hopf bifurcation is determined. Also, results are obtained from the analytical model for finite amplitude convection for which the flow and heat transfer is presented in terms of the governing parameters of the problem. Numerical solutions of the full governing equations are obtained for a wide range of the governing parameters. A good agreement is observed between the analytical model and the numerical simulations.

1. Introduction

Natural convection in fluid-filled rectangular enclosures has received considerable attention over the past several years due to its wide applications in engineering design of advances technology. These applications span such diverse fields as electronic industry, cooling systems for nuclear reactors, heat exchangers, solar energy collectors, etc. A large cross section of fundamental research on this topic has been reviewed by Catton [1] and Bejan [2]. Most of the previous work has addressed natural convection in a cavity filled with ordinary fluids, having in general relatively low thermal conductivities. Thus, the heat transfer capacity through such systems is relatively limited and it is of importance, from the industrial and energy point of view, to improve it. One way to overcome this draw back is to disperse nano-scale particles in the base flow. The heat transfer characteristics of the resulting mixtures are considerably enhanced due to the presence of the nanoparticles in the fluids that increase significantly the effective thermal conductivity of the latter. This technique was developed by Choi [3] who was the first to introduce the term nanofluid to refer to a fluid in which nanoparticles are suspended. A review of the literature on this subject shows that rather little work has been carried out on natural convection in such nanofluids confined in enclosures.

The first study concerning natural convection of a nanofluid confined in a differentially heated enclosure seems to be due to Khanafer et al. [4]. A comparative study of different models based on the thermophysical properties of copper–water nanofluid is developed and investigated. Their numerical results indicate that the suspended nanoparticles substantially increase the heat transfer rate at any given Grashof number. A heat transfer correlation of the average Nusselt number for various Grashof numbers and volume fraction is proposed by the authors. The same problem was considered by You and Tzeng [5]. The Khanafer et al.'s model was used to investigate the convective heat transfer enhancement in rectangular enclosures filled with an Al_2O_3–water nanofluid. It was also reported that increasing the buoyancy parameter and volume fraction cause an increase in the average heat transfer coefficient. Natural convection heat transfer of nanofluids in a square cavity, heated isothermally from the vertical sides, has been investigated numerically by Ho et al. [6] and Santra et al. [7]. Two different formulas have been considered by [6] for the effective viscosity and thermal conductivity of the nanofluids while the Ostwald-de Waele model for a non-Newtonian shear thinning fluid has been used by [7] to calculate the shear stress. It was found that the uncertainties associated with different models adopted to

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1 http://www.meca.polymtl.ca/convection.

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