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Numerical and analytical solutions for the flow and heat transfer near the equator of an MHD boundary layer over a porous rotating sphere

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ABSTRACT

This paper is primely devoted to numerical treatment and derivation of analytical expressions for the solution of steady, laminar, incompressible, viscous and electrically conducting fluid of the boundary layer flow due to a rotating sphere subjected to a uniform suction and injection through the surface in the presence of a uniform radial magnetic field. The fluid and body rotate either in the same direction or in opposite directions. Particular attention is the vicinity of equator of the sphere for which the governing mean flow and temperature equations are derived using the similarity transformations. These are first solved numerically by a spectral Chebyshev collocation integration scheme and later are treated via the recently popular homotopy analysis method to obtain the exact solutions. The homotopy series are mathematically proven to rapidly converge to the numerical solutions, provided that auxiliary convergence control parameters are selected optimally as described here. The resulting flow pattern, temperature distribution, surface shear stresses, infinite radial velocity and heat transfer characteristics are presented and discussed for the various parameter cases. Explicit formulas are also derived for some parameters of physical significance.

1. Introduction

Mathematical modeling of many fluid mechanics problems arising from technological and industrial applications results in strongly nonlinear partial differential or ordinary equations, generally requiring a numerical approach owing to the fact that the exact solutions are almost impossible to deal with. One such physical problem studied here is the electrically conducting boundary layer flow over a porous rotating sphere, which is analyzed numerically as well as via an effective analytical method when the viscous fluid flow finds itself confined near the equator.

Rotating flows over stationary or rotating bodies are known to have applications in several diverse areas such as geophysical and cosmical fluids, meteorology, gaseous and nuclear reactors. The topic of convection heat transfer from rotating bodies has received considerable attention over the past several decades. The problem finds applications in chemical engineering technologies, aerodynamics and planetary astrophysics. The subject of magnetohydrodynamics (MHD) has also developed in many directions and industry has exploited the use of magnetic fields in controlling a range of fluid and thermal processes. Many studies of the influence of magnetism on electrically-conducting flows have been reported with a plethora of other physical phenomena. In terms of magnetohydrodynamic applications, good examples are from the solar physics involved in structure of a rotating magnetic star, solar cycle and sun spot development.

The rotation of a sphere in a quiescent viscous fluid, with and without inertia, has been the object of several papers. When a sphere rotates in still fluid a flow is induced in which the fluid moves over the outer surface from the poles to the equator and is ejected radially from the equator. The resulting three-dimensional flow was first investigated theoretically by Howarth [1], who made boundary-layer approximations to the steady Navier–Stokes equations and used a series solution to calculate the mean flow. Banks [2] used Howarth's series solution and Manohar [3] and Banks [4] used more accurate finite difference techniques. For the case of a sphere rotating in an ambient fluid, no similarity solution was found in the vicinity of the equator, and the nature of the flow in this region was discussed in several papers as such [4]. Whereas Dennis and Duck [5] solved the full Navier–Stokes problem, the experimental papers of Kohama and Kobayashi [6] reported that the flow exhibits transitional and turbulent regions as well as the laminar region. Hollerbach et al. [7] studied the flow caused by the torsional oscillations of a sphere in a fluid at rest.

For the analysis of flow and heat transfer characteristics in the rotating disk systems, one can refer to the recent book [8]. The analytical mean flow solutions are important in the study of the

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