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Thermal wave propagation in a finite medium irradiated by a heat source with Gaussian distribution in both the temporal and spatial domain

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ABSTRACT

This paper presents an analytical solution of transient heat conduction in a thin film irradiated by symmetrical heat source with Gaussian distribution in both the temporal and spatial domain employing the dual-phase-lag model. Heating is modeled as an internal heat source. Calculations are performed to exhibit various lagging thermal behavior.

1. Introduction

In ultra-fast heat conduction systems, the orders of magnitude of the time and space dimensions are extremely short, and thus in treating such situations Fourier's law fails since it detected infinite speed. One of the most important models to alleviate this unrealistic notion is the dual-phase-lag model (DPL), given by Tzou [1]. It allows either the temperature gradient (*cause*) to precede the heat flux vector (*effect*) or the heat flux vector (*cause*) to precede the temperature gradient (*effect*) in the transient process. Mathematically, this can be represented by

$$\underline{q}\left(\underline{r}, t+\tau_q\right) = -K\,\underline{\nabla} T\left(\underline{r}, t+\tau_T\right) \tag{1}$$

where τ_q is the phase lag (relaxation time) of the heat flux vector, τ_T is the phase lag (relaxation time) of the temperature gradient and K is the thermal conductivity. The equation of energy conservation for such problems is given as:

$$\rho c_p \frac{\partial T}{\partial t} = \underline{\nabla}\cdot\underline{q} + Q. \tag{2}$$

where ρ is the density, c_p is the specific heat at constant pressure, and Q is the heat generation per unit volume. Combining the energy equation (2) with the first order approximation of equation (1) and for a medium of constant properties, we get the $T-$ representation of the dual-phase-lag heat equation as

$$\nabla^2 T + \tau_T \frac{\partial}{\partial t}\left[\nabla^2 T\right] + \frac{1}{K}\left[Q + \tau_q \frac{\partial Q}{\partial t}\right] = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T}{\partial t^2}. \tag{3}$$

where $\alpha = K/\rho c_p$ is the thermal diffusivity.

Recently, considerable interest has been generated toward high-intense and ultra-short lasers in thin films, which have found numerous applications [2]. Tang and Araki [3] study the temperature wave propagation and reflection in finite medium due to Gaussian laser-pulse heating in time and exponentially decaying in the spatial variable. The dual-phase-lag heat equation was used by [4] to generalize macroscopic model in treating the transient heat conduction in finite slabs irradiated by short pulse laser. Analytical solution of the dual-phase-lag heat equation of a symmetrically heated finite medium was derived by [5] for different time characteristics of heat sources capacity. Two-dimensional numerical solution in a rectangular and an axially symmetric system is derived by [6] using finite difference method. The correct modeling for Gaussian distributed spatial heat source for the one-dimensional case was proposed by [7] who gave a crucial scientific definition for the Gaussian source parameters. The temperature wave propagation and reflection in finite medium for symmetrically heating thin film with time dependent laser heat source is obtained by [8] which previously solved numerically by [2].

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