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Scale analysis and accurate correlations for some Dirichlet problems involving annular disc

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ABSTRACT

This paper presents analytical investigations in order to determine the heat flow through an isothermal annular disc. This kind of problem has no explicit analytical solutions. Our goal aims to develop compact and accurate approximants valid regardless of the value of the ratio between the inner and the outer radii of the annular disc. The principle of this approach is based on the scale analysis to determinate the asymptotic behaviours and the use of an efficient correlation method. Very compact and accurate solutions are thus derived. Comparisons of the results obtained with the proposed solutions with some available numerical data show an excellent agreement (the relative difference is less than 1%).

1. Introduction

Dirichlet problems for an annular disc are encountered in various scientists' fields as heat transfer, electromagnetism, electrostatic, etc. These problems are very difficult to solve rigorously due to a singularity on the edges of the annular disc. Many authors have investigated this problem since 1950s. Smythe [1] has performed an ingenious analysis based on the superposition technique to determine the capacitance of a circular annulus. Cooke [2] has developed an analytical study from integral method. The author has solved the obtained integral solution by numerical way and has shown that his results are in excellent agreement with those computed by Smythe [1]. Collins [3] has studied the potential problem for a circular annulus. He performed analytical developments for the axisymmetric case using a superposition technique. The solutions are given under imbricated's Fredholm integral equations, which require iterative numerical solving.

The problem of Dirichlet condition on an annular disc is often investigated in the cylindrical coordinate involving the Bessel's functions. The solutions show three integral equations which correspond to the three parts of the surface (i.e. the inner surface, the annular surface and the outer surface). Some authors call this problem: the triple integral equations. Cooke [4] has proposed different solutions for the triple integral equations. These solutions involve imbricated's Fedholm integral equations as Collins [3]. Fabrikant [5] considered the Dirichlet problem taking into account the non-axisymmetry. The obtained solution is also under Fredholm's integral equation form. The kernel of this integral equations beginning non-singular, it can be solved by an iterative method.

The Neumann problems, for which the annual disc is subject to a flow, when the remainder of the surface is subject to an imposed potential, are also exposed to the same difficulties of solving as the Dirichlet problems, hence the same techniques are used [6].

No general solutions to these problems have been attempted yet.

Other kinds of boundary conditions involving disc contact and interface problems have been studied in the literature [7–14].

In this paper we are interested in two different Dirichlet problems:

(Case 1): the annular disc is subject to a Dirichlet condition, with a uniform temperature, when the remainder is insulated,
(Case 2): the annular disc is isothermal, with zero temperature, when the inner surface is subject to a uniform heat flux and the outer surface is insulated.

We propose a method to determine the resistance (or its inverse: the conductance, and the flow) through the annulus. We first proceed to a scale analysis in order to determine the asymptotic

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