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Free convection stagnation-point boundary-layer flow in a porous medium with a density maximum

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ABSTRACT

The steady free convection boundary-layer flow near a stagnation point in a fluid-saturated porous medium is considered when the convecting fluid is close to its maximum density. Three forms for the wall boundary condition are treated, a prescribed wall temperature, prescribed wall heat flux and Newtonian heating. In each case the flow and heat transfer characteristics are determined by a dimensionless parameter δ that measures the difference between the ambient temperature and the temperature at which the fluid attains its density maximum. We find that solutions are possible for $\delta \geq 0$ for each case. For $\delta < 0$ there is a critical value δ_c of δ, the value of which depends on the boundary conditions applied, with solutions possible only for $\delta \geq \delta_c$. The nature of this critical value, as well as other limiting asymptotic forms is discussed.

1. Introduction

Free convection boundary-layer flows arising from density gradients within a fluid-saturated porous medium in a gravitational field are the subject of much ongoing research in the more general area of fluid mechanics and heat transfer because of their important applications in environmental, geophysical and energy related engineering problems, see [1–3] for examples. To analyse these flows the Boussinesq approximation [2] is commonly used together with a linear density–temperature relation. Water, as well as several metals, has its maximum density in the liquid phase. For example, pure water at atmospheric pressure attains its density maximum at about a temperature of $T_m = 3.98^o$ C and this density reversal for lower temperatures can have significant effects on any buoyancy-driven flow. Goren [4] proposed a new relation in which the density difference varies with the square of the temperature difference for those cases where the usual linear density–temperature relation is not adequate. Gebhart and Mollendorf [5] developed a more accurate density relationship for water around the density extremum condition for different salinity levels. Using this relation [6] they were able to find similarity solutions for two-dimensional boundary-layer flows induced by the buoyancy effects of thermal and saline diffusion.

Kay *et al.* [7] analysed the thermal bar, a descending planar plume of denser fluid at temperature T_m in a lighter fluid at temperatures above or below T_m, as a laminar free convection boundary-layer flow, using the density relation proposed by Goren [4]. More recently Cayley and McBride [8] studied the free convection flow in a vertical cylinder of water in the vicinity of the density maximum at about 4 °C both experimentally and theoretically using several density–temperature relations. One of their density–temperature relations, and the one that we shall consider in this paper, has a parabolic variation in temperature T and is claimed [8] to be valid for the range of temperatures from 0 °C to 100 °C. In [8] they also present experimental evidence for the formation of a rising vortex of water, starting in the lower edge regions of the cylinder.

In this paper we consider how a density maximum can affect free convection boundary-layer flows within a fluid-saturated porous medium. We model the flow in the porous medium by Darcy's law and take the density–temperature relation suggested by Cayley and McBride [8]. We also restrict our attention to the flow near a lower stagnation point. This simplification of the flow geometry allows the steady problem to be reduced to similarity form, which we can then treat in detail thus enabling the specific features of a density reversal to be clearly brought out. We treat three separate types of boundary condition, namely a prescribed wall temperature, a prescribed wall heat flux and Newtonian

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