A numerical approach to obtain the yield curves with different risk-neutral drifts

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ABSTRACT

In this paper we consider the possible dependence of the market price of risk on time and interest rates. This fact gives as a result that the risk-neutral drift, which is one of the coefficients of the pricing equation, also depends on time and interest rates. Then, we estimate the risk-neutral drift directly from the slope of the yield curve. This approach is very accurate as we show with a numerical experiment. In order to obtain the term structure we also propose a suitable finite difference method, which converges to the true solution. Finally, we obtain and compare the yield curves with data from the US Treasury Bill market.

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1. Introduction

The term structure of interest rates has fascinated generations of researchers, [1–4]. This fact should not come as a surprise. An understanding of the stochastic behavior of interest rates is important for the conduct of the monetary policy, the public debt management, the expectations of the real economy activity and inflation, the risk management of a portfolio of securities, and the valuation of interest rate derivatives, [5].

This paper focuses attention on one-factor short rate models. Although they have several shortcomings, they are still very attractive for academics and practitioners. They promise to offer stable and consistent models, with parsimonious structure for the fundamental behavior of interest rates and term structure.

In the empirical implementation of the one-factor models there is only one state variable, the instantaneous interest rate. However when we use the Theory of Arbitrage we also need to use the risk-neutral probability or equivalently, the market price of risk which is unobservable. Traditionally, this function has been considered arbitrary and even constant to find a closed-form solution. However, this fact can lead to misspecification.

The aim of this paper is to analyze the effect on the market price of risk dependence on time and interest rate on the yield curves. The cost of considering more realistic functions in the model is that a closed-form solution is not known. However this is not a problem because we propose an efficient numerical method to provide an accurate approximated solution for the term structure problems. Moreover, we propose to estimate the short rate risk-neutral drift directly from the slope of the yield curve. Therefore, we have to estimate neither the interest rate drift nor the market price of risk. This fact reduces the misspecification of these functions, the computational cost of the model and finally, the term structure errors.

The rest of the paper is organized as follows. Section 2 briefly describes a term structure model with one stochastic variable, the instantaneous interest rate. Then, we show some of the most well-known models in the term structure literature, [2,6]. Moreover we propose some generalizations for the market price of risk of these models. As a consequence,

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