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Nonlinear juvenile predation population dynamics ☆

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ABSTRACT

A general nonlinear age-structured predator–prey model is analyzed to obtain the dynamics of two interacting populations that includes self-limitation on the prey and juvenile predation. Our aim is to identify mechanisms of newborn survival that allow us to explain viable interactions between the two populations in circumstances when their absence would otherwise result in unstable behavior with unbounded oscillations. To achieve our goal we apply some standard methods in the analysis of dynamical systems such as the Painlevé property and bifurcation analysis.

1. Introduction

Age structure in predator–prey models has been widely studied in the literature (see, for example, [1–3]). Unstable periodic solutions are frequently found in these models, although they do not correspond to the biological reality where prey and predator populations can coexist. Amongst the efforts to reach coexistence is the inclusion of diverse factors such as self-limitation of the prey, harvesting of prey and cannibalism of the younger prey; see, for example, [4–6].

One of the successful tools used to analyze biological models is the Painlevé property, which has its origin in the work of Kovalevskaya, who was the first to consider the analytical theory of differential equations in physical problems [7]. Paul Painlevé took these ideas and classified ordinary differential equations (ODEs) of second order according to the type of singularities of their solutions [8]. Since then, the property has been used to construct symmetries, to find explicit solutions, to detect control parameters, and so on [9,10].

Basically, a system of ODEs has the Painlevé property if its general solution has no movable critical singular points. Equivalently, we might say that the only singularities of the system of ODEs are poles. Ablowitz et al. [11] described an algorithm named the P-test, which allows one to determine in three steps if an ODE has the property: finding (a) the dominant behavior, (b) the resonances and (c) the constants of integration. It is important to remark that this algorithm does not identify essential singularities. Only necessary conditions for a system to have the Painlevé property are established.

The goal of this paper is to use the Painlevé property in age-structured predator–prey models as a modeling tool in order to provide candidates to stabilize such models and to discover essential biological factors. The main idea is to analyze a general form of the term that models the incorporation of the newborn prey into the dynamics. Finally, to verify coexistence between predator and prey, we will use standard techniques in bifurcation analysis.

2. The model

In this section, we develop an age-structured model that includes self-limitation of the prey and juvenile predation; for details, see [4].

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