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Stability of delay integro-differential equations using a spectral element method

Firas A. Khasawneh *, Brian P. Mann

Mechanical Engineering and Materials Science Department, Duke University, Durham, NC 27708, United States

A R T I C L E I N F O

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This paper describes a spectral element approach for studying the stability of delay integro-differential equations (DIDEs). In contrast to delay differential equations (DDEs) with discrete delays that act point-wise, the delays in DIDEs are distributed over a period of time through an integral term. Although both types of delays lead to an infinite dimensional state-space, the analysis of DDEs with distributed delays is far more involved. Nevertheless, the approach that we describe here is applicable to both autonomous and non-autonomous DIDEs with smooth bounded kernel functions. We also describe the stability analysis of DIDEs with special kernels (gamma-type kernel functions) via converting the DIDE into a higher order DDE with only discrete delays. This case of DIDEs is of practical importance, e.g., in modeling wheel shimmy phenomenon. A set of case studies are then provided to show the effectiveness of the proposed approach.

The evolution of the states in a system are often governed not only by the current states, but also by the past ones. These systems are described mathematically by delay differential equations (DDEs) where the delay leads to infinite dimensional state-space. When the delay is a discrete scalar that acts at a single instant of time it is called a concentrated or discrete delay. However, in many applications the effect of the delay is distributed over a time interval according to the delay integro-differential equation (DIDE)

$$\dot{x}(t) = f\left(x(t), \int_0^{\vartheta} K(t,s)x(t-s)\mathrm{d}s\right), \tag{1a}$$

$$x(t) = \varphi(t) \quad \text{for } t \in [-\vartheta, 0], \tag{1b}$$

where $x \in \mathbb{R}^n, f : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is continuously differentiable, φ is continuous and it represents the history segment, $K : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^{n\times n}$ is a bounded analytic kernel function while $\vartheta > 0$ is the duration of the bounded distributed delay. Examples where several variations of Eq. (1) appear include wheel shimmy models [1–3], traffic models [4–8], machining dynamics [9,10], and neural modeling [11]. A common interest in all these application areas is the stability of equilibria in the governing DIDE. More specifically, in this study we are interested in the stability of Eq. (1) where f is linear in both $x(t)$ and $x(t-s)$.

The stability analysis of DIDEs is more complicated than delay differential equations (DDEs) with discrete delays due to the added complexity of the integral term. This is evidenced by the limited number of studies on DIDEs compared to the more extensive literature on DDEs with discrete delays. Nevertheless, several methods have been used in literature for studying DIDEs.

For instance, the stability conditions of Runge–Kutta type methods for DIDEs were investigated in Refs. [12,13]. Baker and Ford studied the stability of scalar DIDEs with a constant kernel using numerical schemes based on the Newton–Cotes formula combined with a linear multi-step method [14].

* Corresponding author. Tel.: +1 919 660 5338; fax: +1 919 660 8963.
E-mail addresses: firas.khasawneh@duke.edu (F.A. Khasawneh), bpm4@duke.edu (B.P. Mann).

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