Mathematical and Computer Modelling 54 (2011) 2486–2492

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Strong convergence of the Mann iteration for α-demicontractive mappings

Laura Măruşter [a], Ştefan Măruşter [b,*]

[a] *University of Groningen, The Netherlands*
[b] *West University of Timişoara, Romania*

ARTICLE INFO

Article history:
Received 17 February 2011
Received in revised form 2 June 2011
Accepted 2 June 2011

Keywords:
Mann iteration
Strong convergence
Demicontractive mappings

ABSTRACT

The paper deals with strong convergence properties of the Mann iteration. A new class of demicontractive mappings (called α-*demicontractive*) is introduced for which the strong convergence of the computed sequence is assured. The paper presents also an overview of relevant contributions of the last two decades, concerning strong convergence for Mann-type iteration of demicontractive mappings.

1. Introduction

Let $\mathcal{H}$ be a real Hilbert space with inner product $\langle ., . \rangle$ and induced norm $\|.\|$ and let $T : \mathcal{C} \to \mathcal{C}$ be a nonlinear mapping, where $\mathcal{C}$ is a closed convex subset of $\mathcal{H}$. We will suppose throughout the paper that the set of fixed points of T is nonempty, $\mathrm{Fix}(T) \neq \emptyset$.

Definition 1 (*[1]*)**.** The mapping T is said to be demicontractive (or demicontractive with constant k) if

$$\|Tx - p\|^2 \leq \|x - p\|^2 + k\|x - Tx\|^2, \quad \forall (x, p) \in \mathcal{C} \times \mathrm{Fix}(T). \tag{1.1}$$

Usually, the constant k is assumed to be in the interval $(0, 1)$, but often this restriction is avoided; for example, in [2] is considered a class of mappings satisfying (1.1) with negative values of k and called *strongly attracting maps*; a map T satisfying (1.1) with $k = 1$ is called *hemicontractive*, and this condition was used in [3,4] to prove the strong convergence of the implicit Mann iteration introduced in [5].

Definition 2 (*[6]*)**.** The mapping T is said to satisfy condition (A) if

$$\langle x - Tx, x - p \rangle \geq \lambda \|x - Tx\|^2, \quad \forall (x, p) \in \mathcal{C} \times \mathrm{Fix}(T), \tag{1.2}$$

where λ is a positive real number.

Note that (1.2) is a condition of accretive (or monotone) type.

* Corresponding author.
E-mail addresses: l.maruster@rug.nl (L. Măruşter), maruster@info.uvt.ro (Ş. Măruşter).

0895-7177/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2011.06.006